WS-CFT

WS SUSY

X^μ $\quad c = D$

b, c $\quad c = -26$

$\to D = 26$

(BRST)

ψ^μ $\quad c = \frac{1}{2} D$

β, γ $\quad c = 11$

$\to D = 10$

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b, c $\quad c = -26$

β, γ, b, c $\quad c = -15$

X^i $\quad c = 16 + 6$

X^i, ψ^i $\quad c = 9$

X^μ $\quad c = 4$

X^μ, ψ^μ $\quad c = 6$

Superstrings: type I, II, Het

WS - boom

$$X = X_L + X_R$$

$$\partial X(z)\,\partial X(w) = \frac{-1/2}{(z-w)^2} + \dots$$

$$T(z) = -\tfrac{1}{2} :\partial X(z)\,\partial X(z):$$

$$T(z)\,\partial X(w) = \frac{1\,\partial X(w)}{(z-w)^2} + \frac{\partial^2 X(w)}{z-w} + \dots$$

$$\rightarrow h = 1$$

$$T(z)\,T(w) = \frac{1/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} + \dots$$

$$c = 1$$

$$S = \frac{1}{4\pi}\int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\,\partial_\alpha X\,\partial_\beta X$$

$$\rightarrow \frac{1}{4\pi}\int d^2 z\, \partial X\,\bar\partial X$$

$$\partial\bar\partial X = 0$$

$$[\alpha_m, \alpha_n] = m\,\delta_{m+n,0} \qquad \alpha_m \sim \oint \frac{dz}{2\pi i} z^m\,\partial X$$

Continuation of MW to Eucl. sign.

WS Fermions

"real" (Majorana)

$$\psi(z)\psi(w) = \frac{1}{z-w} + \dots$$

odd in $(z-w)$

$$T(z) = -\tfrac{1}{2} :\psi\partial\psi:$$

$$\Rightarrow h = 1/2, \quad c = 1/2$$

$\{\psi_m, \psi_n\} = \delta_{m+n,0}$

$$\bar\partial\psi = 0$$

$$S = \frac{1}{4\pi}\int d^2z\,(\psi\bar\partial\psi + \bar\psi\partial\bar\psi)$$

Complex (Dirac)

$$\psi^{\pm} = \frac{1}{\sqrt{2}}(\psi_1 \pm i\psi_2)$$

odd

$$\psi^{+}(z)\psi^{-}(w) = \frac{1}{z-w} = \psi^{-}(z)\psi^{+}(w)$$

$$T(z) = -\tfrac{1}{2} :\psi^{+}\partial\psi^{-}: + \tfrac{1}{2} :\psi^{-}\partial\psi^{+}:$$

$$h = 1/2, \quad c = 1$$

$$S = \frac{1}{4\pi}\int d^2z\,(\psi^{+}\bar\partial\psi^{-} + \psi^{-}\bar\partial\psi^{+})$$

half-integer weight

$$\varphi(z)(dz)^{1/2} = \varphi(w)(dw)^{1/2}$$

what is $(dz)^{1/2}$?

$$\varphi(w) = \left(\frac{dw}{dz}\right)^{-1/2} \varphi(z)$$

$\Uparrow$ Transition fct.

weight 1

$$\varphi(z)\, dz = \varphi(w)\, dw$$

$$\varphi(w) = \left(\frac{dw}{dz}\right)^{-1} \varphi(z)$$

Transition fcts of $T^*\Sigma$ exist!

$S \otimes S = T^*\Sigma$; S: Spinbdl

(all vector bdl's here are line bdls, i.e. one-dim)

Transport $\left(\frac{dw}{dz}\right)^{1/2}$ of S

an $\sqrt{\cdot}$ of Transitions $\frac{dw}{dt}$ of $T^*\Sigma$

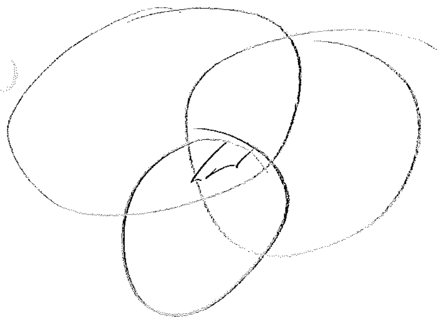
globally consistent $\to \exists\, S$?

unique $\to \exists!\, S$?

$\mathcal{O} = \{ O_a \}$ open cover

$O_{ab} = O_a \cap O_b$

$O_{abc} = O_a \cap O_b \cap O_c$ etc



$t_{ab}\, t_{ba} = 1$

$t_{ab}\, t_{bc}\, t_{ca} = 1$

$\Downarrow \sqrt{\cdot}$

$s_{ab}\, s_{bc}\, s_{ca} = \pm 1 = w_{abc}$

need $c_{ab} = \pm 1$ s.t.

$$c_{ab}\, c_{bc}\, c_{ca} = w_{abc}$$

then $\tilde{s}_{ab} = s_{ab}\, c_{ab}$

and consistent

$w_2 = [\{w_{abc}\}] \in H^2(\mathcal{O}, \mathbb{Z}_2)$ Čech cohomology
(2nd Stiefel Whitney class)

Σ_g: $w_2 = \chi \bmod 2 \simeq 0$ ✓ $\exists$

solution $[\{c_{ab}\}] \in H^1(\mathcal{O}, \mathbb{Z}_2)$

"good cover" $\overset{\sim}{=} H_1(\Sigma_g, \mathbb{Z}_2) \simeq \mathbb{Z}_2^{2g}$

simplicial: $H_1(\Sigma_g, \mathbb{Z}) \simeq \mathbb{Z}^{2g}$

periodic or antiper.

2^{2g} "spin structures" (along gen. of $H_1(\Sigma_g, \mathbb{Z}_2)$)

General 1^{st} order system
"bc-system"

$$c(z)\,b(w) = \frac{1}{z-w} = \varepsilon\; b(z)\,c(w)$$

$$\boxed{\varepsilon = +1\ (F) \qquad \varepsilon = -1\ (B)}$$

$$\bar\partial b = \bar\partial c = 0$$

$$S = \frac{1}{2\pi}\int d^2z\; b\,\bar\partial c$$

$$\boxed{[b] = \lambda \qquad [c] = 1-\lambda}$$ weight

$$\lambda \in \tfrac{1}{2}\mathbb{Z}$$

$$T(z) = -\lambda :b\partial c: + (1-\lambda) :\partial b\, c:$$
$$= \tfrac{1}{2}\left(:\partial b\, c - b\partial c:\right)$$
$$+ \tfrac{1}{2}\varepsilon Q\,\partial :bc:$$

Bagdilly $\quad Q = \varepsilon(1-2\lambda)\ , \quad C = \varepsilon(1-3Q^2)$

$$b(z) = \sum_{n \in a-\lambda+\mathbb{Z}} z^{-n-\lambda} b_n$$

$$c(z) = \sum_{n \in a+\lambda+\mathbb{Z}} z^{-n-(1-\lambda)} c_n$$

For $\lambda \in \mathbb{Z} + \frac{1}{2}$ $\quad a = \begin{cases} 0 & \text{periodic (NS)} \\ \frac{1}{2} & \text{antip (R)} \end{cases}$

then $\quad b_n^{+} = \varepsilon b_n \quad c_n^{+} = c_n$

$$[c_m, b_n]_{\varepsilon} = \delta_{m+n,0}$$

~~$b_n |0\rangle = 0, \quad n \geq 1-\lambda$~~

~~$c_n |0\rangle = 0, \quad n \geq \lambda$~~

~~($\rightsquigarrow$ zero modes for $\lambda < 0$)~~

postponed

Examples

	λ	$1-\lambda$	ε	Q	c
ψ^+, ψ^-	$1/2$	$1/2$	$+1$	0	1
b, c	2	-1	1	-3	-26
β, γ	$3/2$	$-1/2$	-1	2	11

(Zero mode example:

$c_n|0\rangle = 0 \qquad n \geq 2$

$\langle 0| c_n^\dagger = 0 = \langle 0| c_{-n}$

$\langle 0| c_1 |0\rangle \neq 0, \quad \langle 0| c_0 |0\rangle \neq 0$

$\langle 0| c_{-1} |0\rangle \neq 0$

Postponed

$b_n|0\rangle = 0 \qquad n \geq -1$

$\langle 0| b_n^\dagger = \langle 0| b_{-n} = 0$ } no zero modes

$\Rightarrow$ $SL(2,\mathbb{C})$ vac unsuitable for bos string.)

Ghost current

$c \to e^{i\alpha} c \quad , \quad b \to e^{-i\alpha} b$

$$j(z) = - :b(z)\, c(z):$$
$$= \sum_n z^{-n-1} j_n$$

$$j_n = \sum_m \epsilon :c_{n-m} b_m:$$

$$Q^{(j)}(b) = -1, \quad Q^{(j)}(c) = 1$$

$$j(z)\, j(w) = \frac{\epsilon}{(z-w)^2} + \dots$$
$$[j_m, j_n] = \epsilon\, m\, \delta_{m+n,0}$$

$$T(z)\, j(w) = \frac{Q}{(z-w)^3} + \frac{j(w)}{(z-w)^2} + \frac{\partial j(w)}{z-w}$$

anomaly for $Q \neq 0 \iff \lambda \neq \tfrac{1}{2}$

$$\langle \nabla^z j_z \rangle = \frac{1}{4} Q R_h \neq 0 \text{ for } Q \neq 0$$

(p. 39? BLT! bosonization ...)

c.f. $\langle \nabla^\mu T_{\mu\nu} \rangle \sim c R[h]$

$$[L_m, j_n] = -n\, j_{m+n} + \tfrac{1}{2} Q\, m(m+1)\, \delta_{m+n,0}$$

$$j_n^+ = -j_{-n}$$

j_0 has normal ordering ambiguity (cf. L_0)

$$j_0^+ = \left(-[L_1, j_{-1}]\right)^+ = \dots$$

$$= -j_0 - Q$$

Let O has ghost charge p

$$[j_0, O] = p\, O$$

$$p \langle q' | O | q \rangle = \langle q' | [j_0, O] | q \rangle$$

$$= \dots = (-q' - q - Q) \langle q' | O | q \rangle$$

$$\langle q' | O | q \rangle \neq 0 \Rightarrow p = -q' - q - Q$$

$$\text{e.g. } p = 0 \Rightarrow q' = -q - Q$$

$\langle q' | q \rangle \neq 0 \Rightarrow q' = -q - Q$

Normalize $\langle -q-Q | q \rangle = 1$

Bgd Chg, Non-conservation
Anomaly

$$\boxed{q' + q = -Q}$$

Normally charges on a compact Σ_g should add up to zero

$\rightarrow$ extra Bgd chg Q on Σ_g due to anomaly