1. General Theory

1.1. Surveys.

- [Nee21b] is an in-depth survey of the theory. Section 9 could be supplemented with the survey [Nee20].
- [Nee20] is a survey of just [Nee18c].
- [Nee23] is another survey—it focuses on the analogy with Fourier analysis. This survey is less in-depth than [Nee21b]+[Nee20], but it contains a lot of open problems and suggested areas of research.

1.2. Papers.

- [Nee21a] shows $D^{\mathrm{perf}}(X)$ and $D^b_{\mathrm{coh}}(X)$ are strongly generated under relatively weak assumptions. It was the paper that started approximable categories.
- [Nee18a] shows a bunch of natural triangulated categories are approximable (e.g. $D_{\mathrm{qc}}(X)$ when X is quasicompact and separated, the homotopy category of spectra, and $D(R)$ where R is a negatively graded DG-algebra).
- [Nee18b] shows any homological functor (with some reasonable assumptions), $H^i : D^b_{\mathrm{coh}}(X) \to \mathrm{Mod}(R)$ is represented by a perfect complex over X.
- [Nee18c] shows $D^{\mathrm{perf}}(X)$ and $D^b_{\mathrm{coh}}(X)$ determine each other (or more generally $\mathcal{T}^c$ and $\mathcal{T}^b_c$ determine each other).
- [BNP23] shows a recollement of two approximable categories is also approximable.

2. Calculating Completions of Triangulated Categories

2.1. Papers.

- [Nee18c] calculates a bunch of completions
- [Mat24] calculates all possible completions of $D^b(\mathrm{Mod}(A))$ where A is a finite dimensional hereditary k-algebra.
- [CG24] calculates completions with respect to the "aisle" metric.

3. Obstructions to Bounded t-Structures

3.1. Surveys.

- [Nee22b] is a survey, but it is out of date.

3.2. Papers.

- [Nee22a] proves the following:

  Theorem 3.1. *Suppose X is a finite-dimensional, Noetherian scheme and $Z \subseteq X$ is a closed subset. The category $D^{\mathrm{perf}}_Z(X)$—of perfect complexes on X whose cohomology is supported on Z—admits a bounded t-structure if and only if Z is contained in the regular locus of X. In particular, $D^{\mathrm{perf}}(X)$ admits a bounded t-structure if and only if X is regular.*

- [BCR+23] generalizes Theorem 3.1 to

  Theorem 3.2. *Suppose $\mathcal{T} = \langle X \rangle$ is an essentially small, classically generated triangulated category. Assume $\mathcal{T}^{\mathrm{op}}$ has finite finitistic dimension (see* [BCR+23, Definition 1.3]*). $\mathcal{T}$ admits a bounded t-structure if and only if $\mathcal{T}$ is complete with respect to X.*

  [BCR+23] also shows that in the setting of Theorem 3.2 all bounded t-structures are equivalent.
- [Nee18d] gives a criterion for when a "complete aisle" is part of a t-structure.

4. Intrinsic Subcategories

4.1. Surveys.

- [CNSb] is a survey. I think it is up-to-date (as of September 2024).

4.2. Papers.

- [Nee18c] shows $D^{\mathrm{perf}}(X)$ and $D^b_{\mathrm{coh}}(X)$ determine each other.
- [CNSa] shows the categories $\mathcal{T}$, $\mathcal{T}^{\pm}$, $\mathcal{T}^b$, $\mathcal{T}^c$, $\mathcal{T}^-_c$, $\mathcal{T}^b_c$, and $\mathcal{T}^{c,b}$ all determine each other (and are all intrinsically defined). I think a very cool corollary is that the singularity category is a derived invariant. I don't think it was even known that regularity was a derived invariant, but I would need to spend more time looking into this.

5. Strong Generation

5.1. Surveys.

- [Min20] is a survey. I think it is out-of-date. The survey is also really difficult to read if you're not a homotopy theorist.

5.2. Papers.

- [Nee21a] is the main result in this direction. It shows

  Theorem 5.1. *Suppose X is a quasicompact, separate scheme. The category $D^{\mathrm{perf}}(X)$ is strongly generated if and only if X admits an open affine cover, $U = \mathrm{Spec}(R_i)$, where each R_i has finite global dimension.*

  and

  Theorem 5.2. *Suppose X is a Noetherian, separated scheme such that every closed subscheme admits a regular alteration (e.g. X is essentially of finite type over a field). The category $D^b_{\mathrm{coh}}(X)$ is strongly generated.*

- [Aok21] removes the technical assumption on alterations in Theorem 5.2:

  Theorem 5.3. *If X is a quasicompact separated quasiexcellent scheme of finite dimension then $D^b_{\mathrm{coh}}(X)$ is strongly generated.*

- [DDLR24] considers the noncommutative analogue of Theorem 5.1.

6. GAGA Theorems

6.1. Surveys.

- The main idea is contained in [Nee23, Section 6].

6.2. Papers.

- [Hal23] proves a categorical GAGA theorem, but he did not have approximable categories at his disposal.
- [Nee18b] is supposed to bypass most of the technical material in [Hal23]. [Nee18a, Appendix A] shows how to do this.

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