

# DEFINING CONSISTENT ADS-33–METRICS FOR AGILITY ENHANCEMENT AND STRUCTURAL LOADS ALLEVIATION

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## ABSTRACT

The aim of the present paper is to develop a new set of consistent, complementary metrics for agility and structural loads pertaining to vertical manoeuvres in forward flight. In this context, the ADS-33 low-speed attitude quickness parameter is converted into two new metrics: 1) *agility quickness (gamma quickness)* defined as the ratio of peak quasi-steady normal acceleration to flight path angle change during vertical manoeuvres, and 2) *vibratory load quickness* defined as the ratio of peak amplitude of vibratory load to flight path angle change. While the agility quickness characterises the helicopter performance during manoeuvring flight, the load quickness is used to quantify the build up of loads in the rotor. The paper presents how the new metrics can be used to optimise the rotor loading without compromising the aircraft manoeuvrability. In this context, open-loop second harmonic cyclic control inputs, superimposed on the primary longitudinal cyclic pitch controls are analysed, resulting in both helicopter performance improvement and vibratory loads reduction. The case of a helicopter flying at high speed with stall on the retreating blade is investigated, this case being particularly sensitive to higher harmonic blade pitch control. We show how the stalled area can be redistributed over the rotor disk by use of second harmonic cyclic pitch and how this transposes into changes in agility and load quickness parameters. While the use of higher harmonic control through the non-rotating swashplate has been the subject of considerable previous research, the current approach is intended to demonstrate how consistent metrics allow for more efficient and insightful multi-disciplinary optimisation.

## NOTATIONS

C amplitude of the second harmonic input (in)  
g gravity acceleration (m/sec<sup>2</sup>)  
 $n_{zpk}^{qs}$  peak quasi-steady normal acceleration in a pull-up manoeuvre (g's)

$\Delta n_{zpk}^{qs}$  peak quasi-steady normal acceleration defined w.r.t. the level flight (g's)  
 $n_{zpk}^{vib}$  peak amplitude in the vibratory components of the hub shears and/or moments (g's)  
R blade radius (m)  
q helicopter pitch rate (deg/sec)  
q<sub>pk</sub> peak attitude in a pull-up manoeuvre (deg)  
Q<sub>0</sub> attitude quickness parameter (deg<sup>-1</sup>)  
Q<sub>γ</sub> agility quickness parameter (deg<sup>-1</sup>)  
Q<sub>γ</sub><sup>\*</sup> agility quickness parameter (deg<sup>-1</sup> s<sup>-1</sup>)  
Q<sub>l</sub> vibratory quickness parameter (deg<sup>-1</sup>)  
U trim forward speed (kn)  
U<sub>T</sub> component of airspeed normal to the tip-path plane (kn)  
U<sub>P</sub> component of airspeed parallel with the tip-path plane (kn)  
w helicopter vertical speed (kn)  
x fraction of blade radius (x = r/R) (-)  
Z<sub>w</sub>, Z<sub>q</sub>, Z<sub>θ1s</sub> heave derivatives (sec<sup>-1</sup>)  
α blade effective incidence w.r.t. the tip-path plane (deg)  
α<sub>1H</sub> angle of attack when only first harmonic input is applied (deg)  
α<sub>2H</sub> angle of attack when first + second harmonic input is applied (deg)  
Δα<sub>2H</sub> increase in the blade angle of attack due to the application of the second harmonic input (deg)  
β flapping angle (deg)  
β<sub>2c</sub>, β<sub>2s</sub> coefficients in Fourier series for flapping angle (deg)  
γ Lock number (-)  
Δγ flight path change in achieved in a pull-up manoeuvre (deg)  
φ phase of the second harmonic input w.r.t. the first harmonic (deg)  
λ normalized inflow through the disc (-)  
μ helicopter advance ratio (-)  
θ helicopter pitch attitude (deg)  
θ<sub>1s</sub> longitudinal cyclic input (deg)

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$\theta_{2c}, \theta_{2s}$	coefficients in the blade cyclic pitch setting created by the application of the second harmonic input (deg)
$\theta_{1H}$	blade cyclic pitch setting when only first harmonic input is applied (deg)
$\theta_{2H}$	blade cyclic pitch setting when first + second harmonic input are applied (deg)
$\Delta\theta_0$	collective pitch angle introduced by the second harmonic input (deg)
$\Delta\theta_{pk}$	pitch angle change achieved in the pull-up (deg)
$\Delta\theta_{min}$	minimum attitude change in pull-up manoeuvre (deg)
$\Omega$	rotor angular velocity (rad/sec)
$\psi_i$	azimuth angles where $\Delta\alpha_{2H}$ achieves a minimum during a complete rotor cycle (deg)

## INTRODUCTION

The Aeronautical Design Standard ADS-33 (Ref 1) is the US Army's helicopter handling qualities specification born from a need 'to turn what tends to be a vehicle of questionable behaviour into a pure thoroughbred'. ADS-33 is essentially a mission-oriented specification, with criteria depending on selected mission task elements, response types, and usable visual cue environments. The strength of ADS-33 lies in the new criteria and metrics introduced in order to maximise mission effectiveness while not compromising safety. Although ADS-33 criteria offer powerful design goals, they concentrate mainly on *performance* of both aircraft and the pilot. And yet, after the designer has optimised the performance, which mainly depends on average forces and moments on the rotor and fuselage, he or she will certainly encounter another level of difficulty, when facing the world of *vibratory forces and moments*. An entirely new set of problems awaits the structural designer, no longer free to modify the quantities that affect the performance, when trying to control vibration levels. In practice, performance is itself often compromised by the need to control and minimise vibration. One of the questions arising when assessing a rotorcraft's handling qualities is: how can one tackle at the same time the aircraft performance and the vibratory loading problem? A new series of methodologies and integrated tests are therefore needed, capable to deal with the multi-disciplinary nature of a complex rotorcraft.

The present paper presents a new approach to multi-disciplinary optimisation by developing a new set of metrics and test techniques drawn from ADS-33, capable of being applied in both agility enhancement and structural load alleviation. The paper concentrates on developing and exercising the new approach to vertical-pitch manoeuvres in forward flight, this type of manoeuvres being a good example on how the agility and structural loading problems link with each other in manoeuvring flight. As helicopter example for the present applications it will be considered the

generic rotorcraft model (UH-60 – type) modelled with FLIGHTLAB (Ref 6 *FGR*). The new metrics will be further used in the design of control functions that utilise multi-harmonic inputs, properly phased for simultaneous performance enhancement and vibratory reduction.

The paper summarises the first phase of a new collaboration between Delft University of Technology and The University of Liverpool in the general area of rotorcraft handling qualities and multi-disciplinary optimisation.

The paper is structured as follows:

- The first section defines the attitude quickness criterion in the pitch axis according to the ADS-33, discussing some of the advantages and disadvantages learned since its introduction as a handling qualities parameter;
- The second section defines the agility and vibratory quickness parameters and illustrates how these new metrics may be used for evaluating rotorcraft agility and vibratory characteristics during vertical manoeuvres in forward flight;
- The third section utilises the new parameters to optimise simultaneously the rotor loading and helicopter performance. In this context, the rotor disc stall patterns in manoeuvring flight at high forward speed will be described and modified. More precisely, the high loading regions will be shifted around the rotor disc azimuth - by the application of an open-loop second harmonic cyclic input, superimposed on the primary longitudinal cyclic. Using a simple pulse input, the research has explored the variations in magnitude and phase of the higher harmonic feathering control that could be used to modify the stall pattern. Results of this exploration are presented, followed by an investigation into how the agility and vibratory quickness charts change with the application of higher harmonic inputs;
- Finally, general conclusions and planned and potential further extension to this work are discussed.

## ATTITUDE QUICKNESS CRITERION IN THE PITCH AXIS IN ADS-33. ADVANTAGES AND SHORTCOMINGS

As mentioned in the Introduction, the paper concentrates on vertical manoeuvres in forward flight. One of the innovative parameters introduced by ADS-33 in order to characterise moderate amplitude manoeuvres in the pitch axis (i.e.  $5 < \theta < 30$  deg) is the pitch *attitude quickness (theta quickness)*. Attitude quickness pertains to the aircraft agility and measures the helicopter's ability to achieve rapid, precise attitude changes when performing a sharp pitch manoeuvre (for more detail see Ref 2). Consider for illustration the kinematics of a manoeuvre in the pitch axis to change aircraft attitude – a simple example of this type is the

helicopter trying to fly over an obstacle when the pilot applies a pulse input in longitudinal cyclic (see Figure 1a).

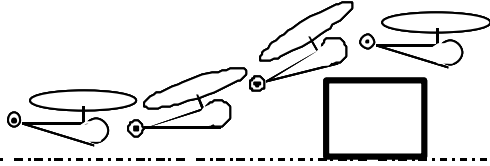


Figure 1a: Executing an obstacle-avoid manoeuvre in the pitch axis

The simulation model used in the current activity was the (UH-60-like) FLIGHTLAB Generic Rotorcraft (FGR), featuring rigid blades, non-linear aerodynamics and a 3-state dynamic inflow model. Figure 1b presents results from the FGR flying with and without its stability and control augmentation system (SCAS), following a 2 second pulse input from the initial trim at 100 kn.

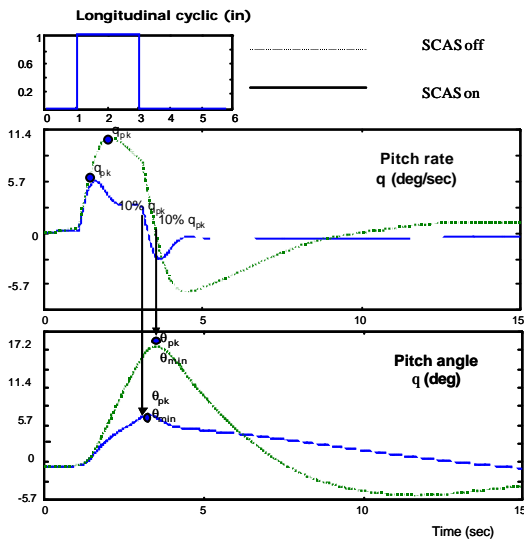


Figure 1b: Helicopter response to pulse longitudinal cyclic input; 100 kn, 1 in input, 2 sec pulse duration

The attitude quickness parameter  $Q_q$  in ADS-33 is defined as the ratio of the maximum pitch rate  $q_{pk}$  to the peak attitude angle change  $\Delta q_{pk}$  that is:

$$Q_q = \frac{q_{pk}}{\Delta q_{pk}} \text{ (sec}^{-1}\text{)} \quad (1)$$

ADS-33 defines handling qualities boundaries for the attitude quickness parameter as a function of the minimum attitude change  $\Delta q_{min}$ . However, this criterion and these boundaries apply only to hover and low speed manoeuvres (<45kn). In forward flight (>45kn), ADS-33 is more

qualitative in terms of flight path handling qualities, and no levels of aggressiveness are defined.

Bearing in mind that the present investigation analyses manoeuvres in forward flight, it was decided to extend the definition of the minimum pitch change  $\Delta q_{min}$  in the ADS-33 attitude quickness criterion as follows: the minimum pitch change  $\Delta q_{min}$  in a pulse manoeuvre in forward flight is the value of pitch angle corresponding to the time to a 10% decay from the maximum pitch rate  $q_{pk}$ . Using this definition, a series of pulse inputs were flown with the FGR model, varying the pulse duration and the initial forward velocity. Figures 2a and 2b illustrate the attitude quickness charts for the pulses flown with SCAS on, and then switching it off. The ADS-33 level 1/2 boundary for a general mission task element is also plotted in order to give the impression how the quickness in forward flight is situated with respect to the ADS-33 level corresponding to low speed manoeuvres.

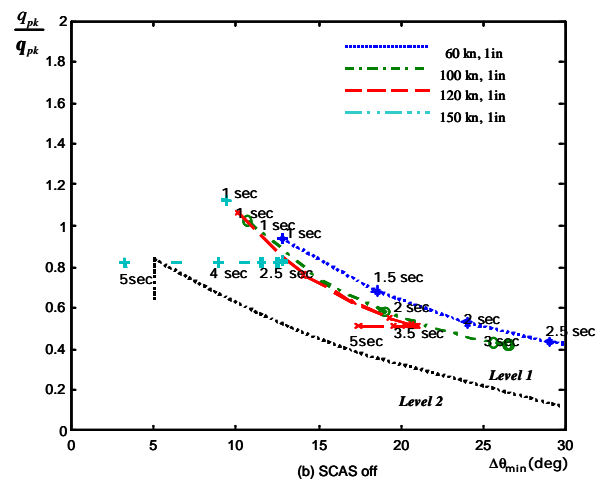
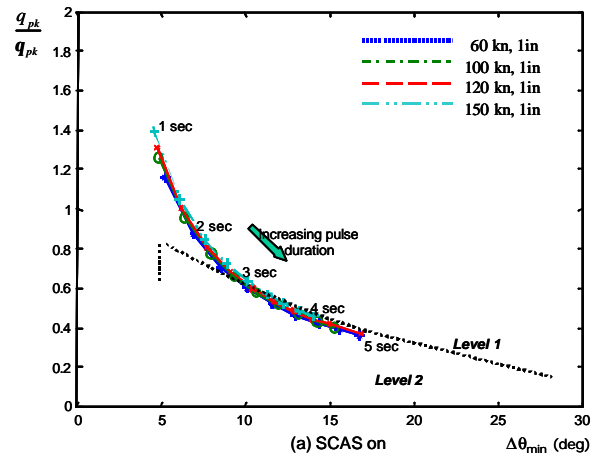


Figure 2: Attitude quickness criterion for pull-up manoeuvres

Looking at these figures, an important conclusion can be drawn. Whereas for the augmented model the attitude quickness follows the characteristic exponential shape as in the ADS-33 - decreasing as the pulse duration increases, for the unaugmented model flown at high forward velocities and increasing the pulse duration, this exponential shape breaks down. This is thought mainly to be due to the hybrid behaviour of the helicopter, combining the characteristics of a rate-command system with the characteristics of an attitude-command system. This causes  $Dq_{min}$  from a certain input to decrease while increasing the pulse duration; meanwhile  $q_{pk}/Dq_{pk}$  remains constant ( $q_{pk}$  and  $Dq_{pk}$  increase achieved a maximum limit and do not increase further as the pulse duration is increased). This problem was reported also by others: DLR at Braunschweig conducting an evaluation of the quantitative forward flight criteria of the ADS-33 by flight testing an unaugmented Bö-105 helicopter reported that it often proved difficult to obtain the minimum bank angles when measuring the attitude quickness in the roll axis (see Ref 3).

Attitude quickness in ADS-33 refers to a rate-command helicopter. For a pure attitude response-type aircraft, the attitude quickness should be based on a step input. The SCAS transforms the hybrid behaviour of FGR into a constant rate-command type throughout the entire flight envelope.

In conclusion, ADS-33 attitude quickness is on the one hand a powerful new tool to measure helicopter agility; nevertheless, on the other hand, attitude quickness is difficult to define for moderate amplitude manoeuvres in forward flight.

This conclusion was underlined also by Bruce Kothmann in his comments on this research (Ref 4): *'There is a fundamental tension which seems to be unrecognised in ADS-33, namely the precision pointing tasks demand a high bandwidth pitch response, while the usual pilot manoeuvring is more about controlling flight path angle with load factor. The tension is that the very high pitch bandwidth required by ADS-33 disrupts the "harmony" with the heave response, in that pitch rate becomes an angle of attack (and therefore load factor) rate command, whereas pilots usually think of it as being directly related to load factor.'* From this tension follows the question whether one is able to define a new metric better able to representing the rotorcraft characteristics on the pitch axis in forward flight.

## DEFINING COMPLEMENTARY METRICS FOR AGILITY AND STRUCTURAL LOAD ALLEVIATION IN VERTICAL MANOEUVRES IN THE PITCH AXIS

### Definition of the agility quickness (gamma quickness) parameter and physical interpretation

It is well-known that the primary functions of pitch control is to provide the mechanism for pulling g in manoeuvres, enabling target tracking by controlling the pitch and flight-path angle. A pilot commands the longitudinal cyclic input in forward flight mainly to change the flight path angle  $g = q - \alpha$  (where  $\alpha$  is the angle of attack of the helicopter). Therefore, during a vertical axis manoeuvring, the pilot is in reality more interested in the flight path angle change  $Dg$  than in the pitch change  $Dq$ . When the flight path angle  $\gamma$  changes so does the normal acceleration – in fact they are closely related.

With these thoughts in mind, the present paper proposes to substitute the conventional ADS-33 attitude quickness parameter for vertical manoeuvres in the pitch axis by a new parameter, the so-called *agility quickness (gamma quickness)* defined as:

$$Q_g = \frac{^{def} n_{zpk}^{qs}}{\Delta g} \quad (2)$$

where  $n_{zpk}^{qs}$  is the peak quasi-steady normal acceleration in g units corresponding to a step change in flight path angle  $\Delta\gamma$ . It can easily be shown that the normal g peak is closely related to  $\dot{g}$ .

Indeed, thinking in terms of pilot performance, *gamma quickness* seems to be a more suitable measure of short-term agility than *theta quickness*. ADS-33 attitude quickness criterion defines boundaries for the theta quickness parameter as a function of the minimum attitude change. The new agility quickness criterion would then be defined by boundaries for the gamma quickness parameter as a function of the flight path change  $\Delta\gamma$  during vertical manoeuvres. The present activity has not sought to define these boundaries, but this is clearly a topic for future research.

Returning to the elementary manoeuvres investigated in the previous paragraph – pulses in the longitudinal cyclic, let us test the representation of the new criterion when manoeuvre aggressiveness varies. As in the attitude quickness, this was accomplished by varying the pulse duration (from 1 to 5 seconds), pulse amplitude (1 in and 2 in pilot input) and the helicopter forward speed (60 kn and 150 kn). Figure 3 presents the agility quickness results using the FGR model and keeping the SCAS on.

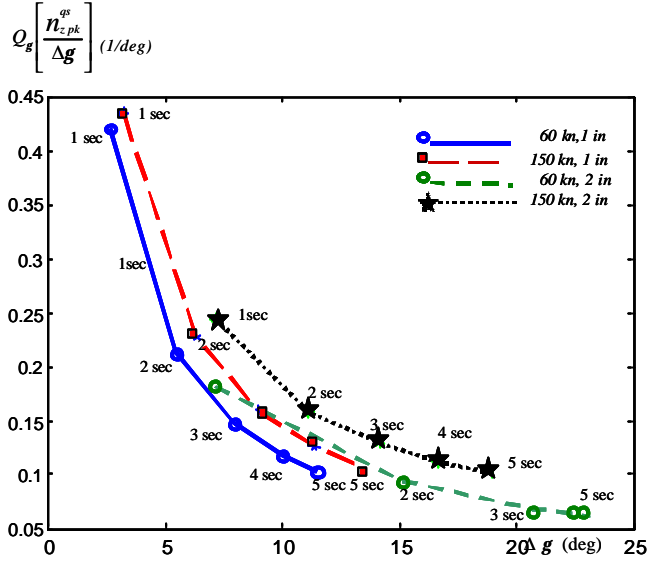


Figure 3: Agility quickness envelope for a pulse in longitudinal cyclic; forward flight SCAS on

As the pulse duration increases the pilot is able to increase the flight path change and pull more g's up to a limit when the rotor stalls.

One may observe that, with SCAS on, the gamma quickness definition is consistent with the ADS-33 attitude quickness criterion and looks much like the classical exponential representation of the theta quickness. It should be mentioned that attempts were made to determine the same chart for unaugmented helicopter. In this case, the exponential shape of the curve was lost when flying at higher speeds and the larger amplitude inputs, as a consequence of the helicopter hybrid response type. Consequently, it was decided to continue the present study for the augmented helicopter only in order to avoid discontinuities in the agility quickness charts.

One of the reasons the ADS-33 attitude quickness criterion has gained large acceptance as a handling quality specification is due to its appealing physical interpretation, where the limiting cases give the time constant of the aircraft as a function of the time constant of the manoeuvre (Ref 2). For the case of attitude quickness, this can also be interpreted as approximating the attitude bandwidth at small amplitude and control power at high amplitude.

What is the physical interpretation of the gamma quickness parameter? In order to gain useful insight into the properties of the agility quickness we start from the heave equation for a helicopter flying with constant forward speed (Ref 4):

$$\dot{w} = Z_w w + Z_q q + Z_{q_s} \mathbf{q}_{1s} + Uq \quad (3)$$

where  $Z_w$ ,  $Z_q$  and  $Z_{q_s}$  are heave derivatives and  $U$  is the trim forward speed. If the pitch response bandwidth meets the ADS-33 forward flight air combat bandwidth requirement (Ref 1), it is reasonable to ignore the pitch rate dynamics (i.e. assume that pitch rate is simply proportional to longitudinal control) when considering flight path manoeuvres. Further, if the moments are dominated by main rotor flapping, the last two terms in the heave equation (3) will balance leading to the following approximation:

$$\dot{w} = Z_w w + Uq \quad (4)$$

The maximum heave velocity achieved from a T seconds longitudinal pulse input is then:

$$w_{\max} = -\frac{Uq}{Z_w} \left(1 - e^{Z_w T}\right) \quad (5)$$

This will lead to a maximum change in load factor  $\Delta n_{zpk}^{qs}$  (the change in load factor  $\mathbf{Dn}_z$  is defined as  $n_z - 1$  w.r.t. the level flight):

$$\Delta n_{zpk}^{qs} = -\frac{Z_w w_{\max}}{g} = \frac{Uq}{g} \left(1 - e^{Z_w T}\right) \quad (6)$$

Scaling the agility quickness parameter  $Q_g$  by  $(g/U)$  and substituting (5) into (1) leads to:

$$Q_g^* = \frac{g}{U} \frac{\Delta n_{zpk}^{qs}}{\Delta g} = \frac{q}{\Delta g} \left(1 - e^{Z_w T}\right) \quad (7)$$

For  $t > T$ , the heave velocity will decay to zero and the final flight-path angle is simply equal to the pitch attitude change  $\mathbf{Dg} = qT$ . Eliminating T in (7) gives the final expression of the scaled agility quickness parameter  $Q_g^*$ :

$$Q_g^* = \frac{g}{U} \frac{\Delta n_{zpk}^{qs}}{\Delta g} = \frac{q}{\Delta g} \left(1 - e^{Z_w \Delta g / q}\right) \quad (8)$$

- For small  $\mathbf{Dg}$ , a first-order Taylor series expansion reveals that  $Q_g^* \approx -Z_w$ ;
- For large  $\mathbf{Dg}$ , ignoring the exponential in (8) and noting that  $\Delta g \rightarrow \Delta q$  gives  $Q_g^* \approx q / \Delta q$ .

Concluding, while the small-amplitude agility quickness is given by heave damping (or heave bandwidth), the large-amplitude agility quickness depends mainly on the quasi-steady pitch rate and thus on the amplitude of the input

(maximum load factor), in the limiting case equivalent to attitude quickness.

### Definition of vibratory load quickness parameter

As stated in the beginning, the aim of the present paper is to define complementary metrics for agility and structural loading during manoeuvring flight. In this context, the question arises as to what complementary metrics could be linked to the agility quickness for structural load alleviation. Concerning the vibratory loads, as the helicopter forward speed increases the vibratory activity of the rotor amplifies and hence, the vibratory hub loads and accelerations increase. This situation is accentuated during manoeuvres. For an N-bladed rotor, the vibratory rotor loads in the blade rotating system transmit as N/rev components in the hub non-rotating system. The paper proposes an equivalent *vibratory load quickness* parameter defined as:

$$Q_l \stackrel{def}{=} \frac{n_{zpk}^{vib}}{\Delta g} \quad (9)$$

where  $n_{zpk}^{vib}$  represents the peak amplitude in g unit's in the vibratory components of the hub shears. An equivalent expression can be defined for hub moments..

This new parameter was tested in a pull-up manoeuvres with different levels of aggressiveness, varying again the pulse duration (1 to 5 second), pulse amplitude (1 and 2 in pilot stick backwards) and helicopter forward speed (60 and 150 kn). Using the FFT and time representations of the hub shears ( $F_{x\ hub}$ ,  $F_{y\ hub}$  and  $F_{z\ hub}$ ) and moments ( $M_{x\ hub}$ ,  $M_{y\ hub}$ ,  $M_{z\ hub}$ ) as resulting from the FGR calculations, it was concluded that, for the analysed pull-up manoeuvres, the critical load is the 4/rev component of the hub vertical shear.

Relating  $n_{zpk}^{vib}$  for the pull-up manoeuvres to the variation in the peak amplitude of the 4/rev, hub vertical shear component, as with the agility quickness charts, we can plot the resulting  $Q_l$  as a function of the flight path angle  $\Delta\gamma$ .

The peak amplitudes of the 4/rev component of the hub vertical shear were calculated as follows:

- each second during the pull-up manoeuvre the max-to-min amplitude (in g unit's) of the 4/rev hub vertical shear component was calculated using the FFT representation,
- the peak amplitude in g unit's of this quasisteady function was extracted,
- the ratio of the peak amplitude to the corresponding flight path change defines the vibratory quickness parameter.

This value characterises the hub vibratory activity in pull-up manoeuvres and can be plotted on a chart as a function of the flight path change  $\Delta\gamma$ . Figure 4 presents the equivalent vibratory load quickness corresponding to the agility quickness represented in Figure 3.

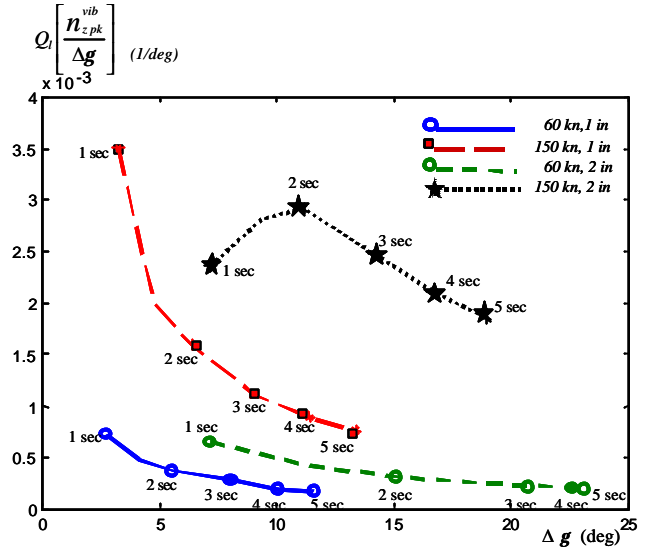


Figure 4: Load quickness envelope for a pulse in longitudinal cyclic; forward flight

One may see that the vibratory load quickness parameter  $Q_l$  varies approximately inversely with the flight path change, decreasing as the pulse duration increases. This is because the vibratory activity in the hub reaches its absolute peak rather quickly, depending mainly on the forward velocity and input amplitude and not on the pulse duration. The only exception to this representation is in the case of 150 kn and 2 inch longitudinal inputs, where for a very short pulse duration (one sec) the absolute peak in vibratory activity couldn't be reached. Questioning the accuracy in modelling the vibratory loads, Ref 5 compared different state-of-the-art models for predicting the hub loads, generally concluding that at low flying speeds vertical shear prediction relies on a good vortex wake model, at high speeds most codes were not able to predict better than 50% of the amplitude of the test results. Concerning the FLIGHTLAB software, FGR uses a finite state wake. The inclusion of the free-wake component in FLIGHTLAB improved considerably the loads correlation, as presented in Ref 5. At high velocities the errors in vibration amplitude prediction in FLIGHTLAB are attributed to a significant phase error in the 3/rev flap moment distribution. The current FGR model includes rigid blades and the simplified wake hence it is not expected that the vibratory load levels will be particularly realistic. In this context, it is emphasised that it was not the intention of this paper to concentrate on the loads prediction accuracy. The exercise of this paper is mainly to present a new approach

for agility and handling qualities specifications, leaving the question of model accuracy outside the study.

### DEFINING CONTROL FUNCTIONS FOR PERFORMANCE ENHANCEMENT AND VIBRATION REDUCTION USING AGILITY AND VIBRATORY LOAD QUICKNESS METRICS

The agility and vibratory load quickness defined above can be used during design in order to optimise the rotor loading without losing aircraft agility. Such an example will be investigated in the following section and relates to a practical problem observed when flying the HELIFLIGHT simulator at Liverpool University (Ref 6). Currently, flying the FGR model in Liverpool, it was observed that at high forward velocities, when executing a rapid pull-up manoeuvres, in order to avoid obstacles, the engine torque (thus also the rotor torque) increases, rather than decreasing as expected. Normally, pulling back the cyclic results in tilting back the rotor disc w.r.t. shaft and increasing the rotor disc angle of attack; as a consequence, the rotor thrust increases and the helicopter starts to climb and in the first seconds of the manoeuvre the rotor rotational velocity increases. This results in an automatic compensation for the rotor torque in order to keep the rotor r.p.m. constant and can be observed by the pilot as an decrease in the engine torque.

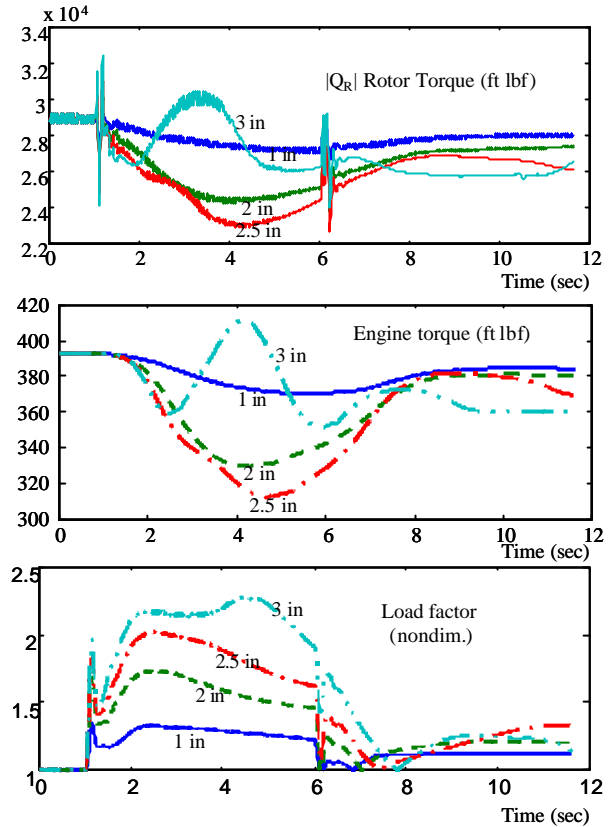
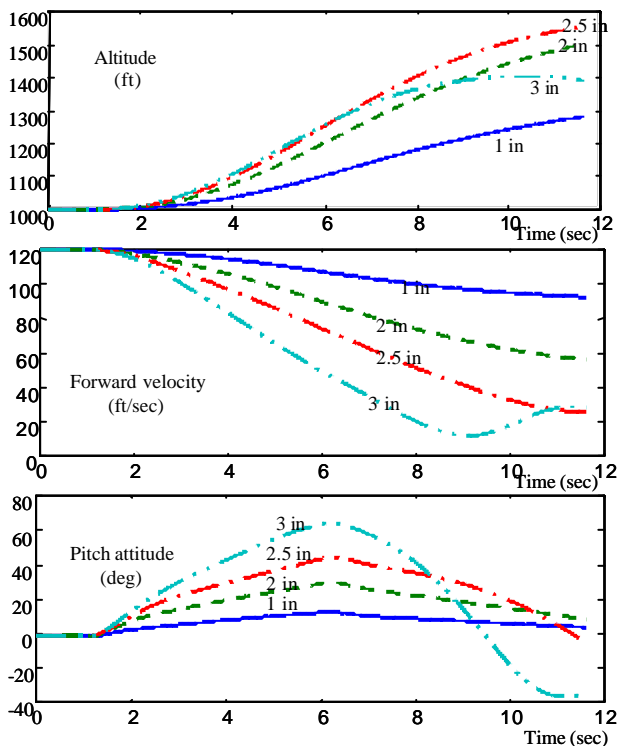


Figure 5: Five second pull-up manoeuvres, 120 kn initial flight

Initially, the detailed modelling of the rotor aerodynamics in the FGR model was reviewed. Performing an off-line analysis it was realised that in fact, in high-speed forward flight during pull-up manoeuvres, the rotor stalled and the performance of the helicopter was consequently degraded. This can be seen in Figure 5 which shows the response to a 2 second pulse input from 120 kn forward flight with the pilot input increasing from 1 to 3 inch. Above an input of 2.5 inch the helicopter starts to descend instead of continuing the climb; this response correlates with an increase in both rotor and engine torque.

The present incident is an indication that the rotor was stalling. The type of stall the rotor encountered was investigated. Analysis of the blade angles of attack, blade lift and drag, showed that the rotor stall patterns (i.e. the angle of attack distribution over the rotor disc at the stalling speed) were limited to a relatively small portion of the rotor disc in the retreating disc area when flying pull-ups in the normal range. However, pull-ups executed when the helicopter is operating near the flight envelope boundary show that the retreating blade stall generalises to a large area, both on the retreating and advancing side of the rotor disc.

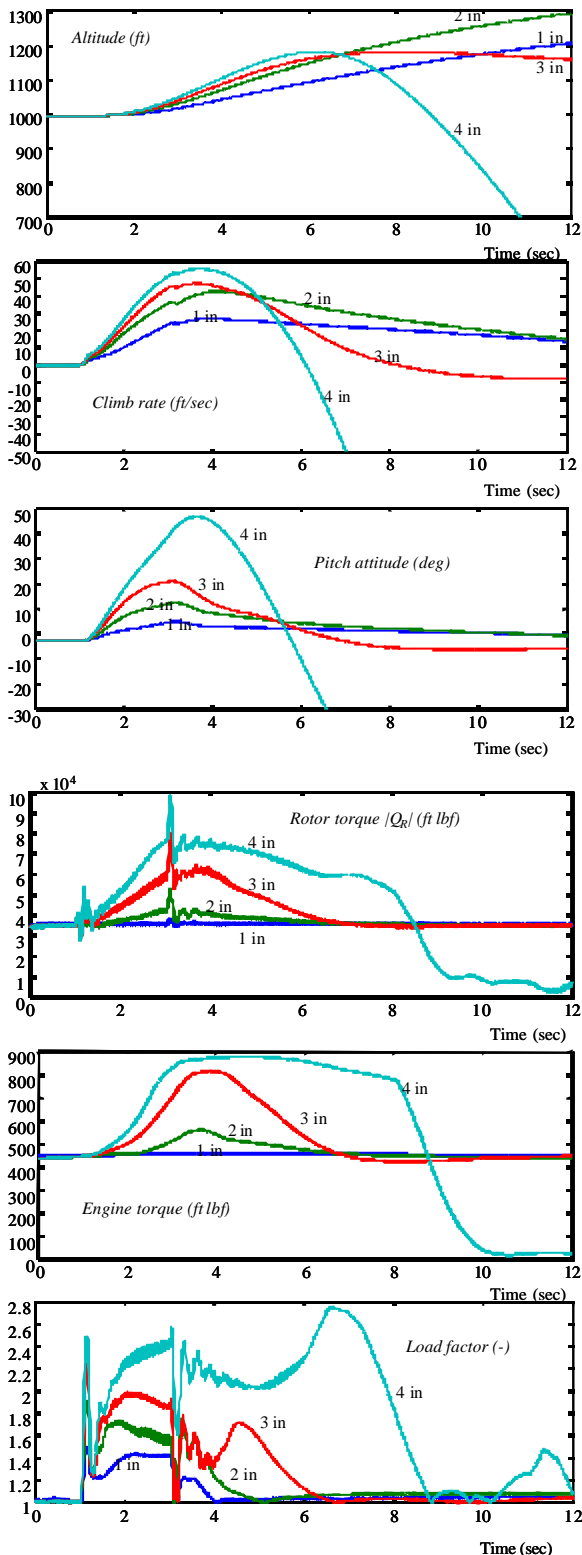


Figure 6: Two second pull-up manoeuvres from 150 kn initial flight

Pulling up with a 2 second cyclic input from 150 kn (Figure 6) initial forward velocity, the blade stalled for longitudinal cyclic pulse inputs greater than 2 inches. Concerning the vibratory loads, when the blade stalls, there is an obvious increase in the higher frequency activity of the blades and hub (see load factor traces in Figure 6).

Summarising these observations, blade stall results in compromising the total aircraft performance and increasing the vibratory activity. Blade stall has always been regarded as a major barrier to improving helicopter forward flight and manoeuvre performance characteristics and much research has been done to aid the understanding of the phenomenon. The classical way in which the stall pattern can be modified consists of the application of a second or higher harmonic pitch control to the rotor blades superimposed on the first harmonic cyclic pitch control.

The following section examines the application of open-loop second harmonic cyclic control inputs superimposed on the primary longitudinal cyclic pitch control when flying at high velocities (150 kn case will be considered) and performing the pull-up manoeuvres simulated in the HELIFLIGHT facility at Liverpool. The objective will be to establish whether, in the case of rotor stalling, the new metrics defined in this paper may be used to optimise loading to increase performance while reducing vibratory loading.

#### OPEN-LOOP SECOND HARMONIC CONTROL INPUTS APPLIED WHEN THE HELICOPTER IS OPERATING NEAR FLIGHT BOUNDARY

The first reported application of second harmonic control on a rotor goes back in 1951 and was carried out in an attempt to introduce vibratory stresses similar to the flight stresses during ground-testing on a rotor tower model (Ref 7). It was soon realised that the use of second harmonic control may be relevant in solving the difficult design challenge of redistributing the loading on the disc to avoid blade stalling conditions limiting helicopter forward speed. Stewart (Ref 8) developed the theory for the second harmonic control and showed for a particular case how the rotor loads can be redistributed by use of this control. He demonstrated that the flapping motion and subsequent incidence distribution depended mainly on blade Lock number  $\gamma$ . For conventional rotor blades, about 70% of the second harmonic control could be regarded as 'useful' incidence. Studying the phase angle between the second harmonic pitch control and the maximum flapping, Stewart concluded that the flapping lags the second harmonic control with a phase angle between 45 and 90 degree. Payne (Ref 9), investigating further the possibility of delaying the stall limit, concluded that the second harmonic feathering alone was not particularly effective in delaying the stall limit. He examined also a suitable application of several higher harmonic inputs to push the retreating stall beyond the advancing blade



compressibility limit. Also, an improvement in helicopter stability was obtained through the application of the second harmonic control. Reference 10 showed that through the application of second and third harmonic control, the maximum speed of a typical helicopter could be increased by 25 to 30 percent. During the 1980's, higher harmonic blade pitch control (HHC) became an attractive solution for controlling the vibratory blade loads and much research has been done for its practical application. Nevertheless, it has been observed that all the HHC flight tests correspond to the helicopter flying well within the normal flight boundary. Reference 11 investigated the application of the higher harmonic blade pitch for a helicopter operating at high speed and thrust. It was demonstrated that using higher harmonic blade pitch may result in an almost completely suppression of the vibratory hub shear components, even though HHC may increase stall areas on the rotor disk.

Consider the FGR model flying at 150kn initial forward velocity and the corresponding agility quickness envelope presented in Figure 3. As discussed above, at 150 kn initial velocity, the rotor retreating blade stall region is expanding on the rotor disc as the pulse duration is increased. If the retreating blade stall pattern can be shifted forwards or backwards along the rotor disc azimuth, then the rotor lifting efficiency would be improved. The first harmonic feathering control cannot be used to modify the disc stall pattern because of the nearly complete cancellation of the desired effects by the blade flapping response. However, higher harmonic controls could be used to modify the stall pattern since the flapping response of the blades to feathering controls of second or higher frequency is small, being well removed from the one-per-rev resonant effect. The most critical issue for the application of second or higher harmonic feathering control concerns the magnitude and phase of the higher harmonic feathering controls to be applied in order to achieve complementary improvements in performance and vibratory characteristics.

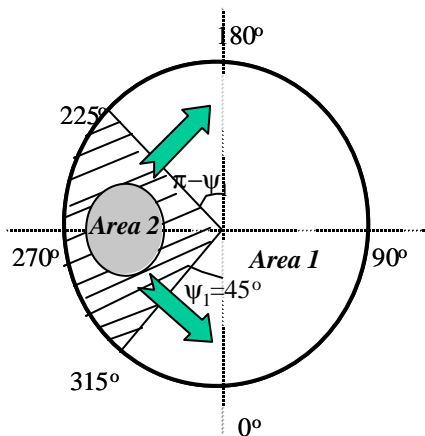


Figure 7: Redistribution of the rotor lift by application of the second harmonic control

Let us define an area on the retreating side of the disc where the angle-of-attack should be redistributed by the use of second harmonic control (see Figure 7) and consider this area defined between  $225 \div 315$  deg azimuth – Area 2. Actually, the angle of attack will be redistributed in the hashed region situated in the second third of the blade radius; inside the rotor disc, the reverse flow effects will result always in a deep stall.

What is the variation of the angle of attack in this region in the initial pull-up manoeuvre? For a 1-second pulse input the angle-of-attack distribution along the rotor disc during the application of the pulse input can be seen in Figure 8a.

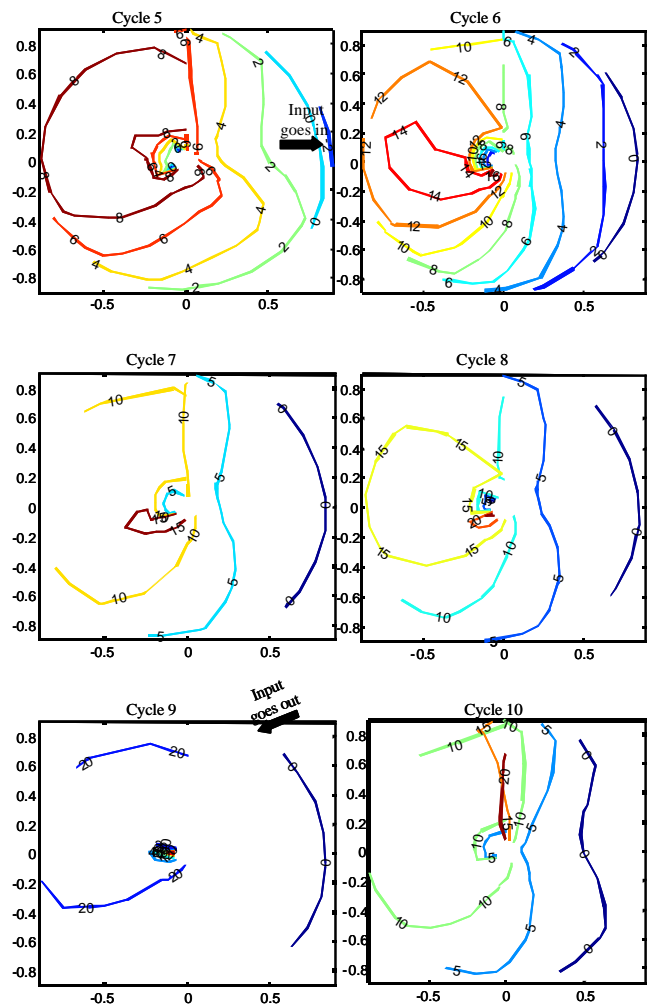


Figure 8a: Stall patterns for a 1 sec pulse, 150 kn

Looking at the retreating side of the disc, we see that the blade incidence in this region increases from 8 deg in cycle 5 to 20 deg in cycle 9. From a blade incidence of higher than 16 deg, one may assume that the blade is stalling (neglecting dynamic stall effects). Thus, from cycle 9 the retreating

blade has reached the stalled condition. The same results were re-plotted in Figure 8b, this time representing only the angle of attack distribution at 0.8R from the rotor centre against the rotor azimuth in each cycle of the manoeuvre.

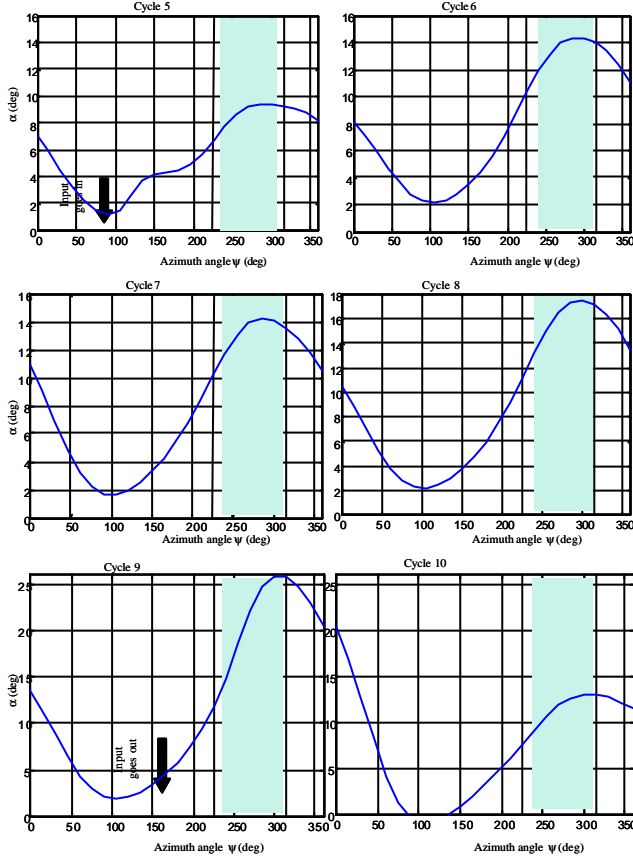


Figure 8b: Variation of the blade angle-of-attack in each cycle during a 1 sec pull-up manoeuvre

This allows us to see more clearly the variation in the incidence in Area 2 of the disc. This part of the disc is important when considering the superposition of second harmonic control inputs. From Figure 8b one may conclude that in each cycle during the manoeuvre, the angle of attack is increasing in this region, reaching its maximum value around cycle 9. It follows that if one is able to apply the second harmonic input so that the resulting change in incidence distribution is out of phase with the initial variation in incidence, the total blade incidence in Area 2 will decrease.

The blade cyclic pitch setting before the application of the second harmonic input for a pulse input of duration  $t_1$  can be expressed as:

$$\mathbf{q}_{1H} = \mathbf{q}_{1s} \sin \mathbf{y}$$

$$\text{where } \begin{cases} \mathbf{q}_{1s} = 0 & 0 < t < 1, \quad t > t_1 \\ \mathbf{q}_{1s} = \text{const.} & 1 \leq t \leq t_1 \end{cases} \quad (10)$$

and after the application of the second harmonic of amplitude  $C$  and phase  $\phi$ :

$$\mathbf{q}_{2H} = (\mathbf{q}_{1s} + C \sin(\mathbf{y} - \mathbf{j})) \sin \mathbf{y} \quad 1 \leq t \leq t_1 \quad (11)$$

Re-arranging the terms in  $\mathbf{q}_{2H}$ , it results:

$$\begin{aligned} \mathbf{q}_{2H} &= \Delta \mathbf{q}_0 + \mathbf{q}_{1s} \sin \mathbf{y} + \mathbf{q}_{2c} \cos 2\mathbf{y} + \mathbf{q}_{2s} \sin 2\mathbf{y} \\ \Delta \mathbf{q}_0 &= \frac{C}{2} \cos \mathbf{j}; \mathbf{q}_{2c} = \frac{C}{2} \cos \mathbf{j}; \mathbf{q}_{2s} = \frac{C}{2} \sin \mathbf{j} \\ &1 \leq t \leq t_1 \end{aligned} \quad (12)$$

Observe that the application of higher harmonic input introduces a collective pitch angle  $\Delta \theta_0$ . The effective incidence of the blade element with respect to the tip-path-plane is given by:

$$\mathbf{a} = \mathbf{q} + \frac{U_P}{U_T} \quad (13)$$

The velocities  $U_P$  and  $U_T$  respectively are normal and parallel to the mean tip-path plane and can be expressed in the system of coordinates of FGR, as:

$$\begin{aligned} U_P &= \left( \mathbf{1} - \mathbf{b} \mathbf{m} \cos \mathbf{y} - \frac{\dot{\mathbf{b}}}{\Omega} x \right) \Omega R \\ U_T &= (x + \mathbf{m} \sin \mathbf{y}) \Omega R \end{aligned} \quad (14)$$

with  $\lambda$  the normalised inflow through the disc,  $\mu$  advance ratio,  $x = r/R$ , the fraction of blade radius and  $\beta$  the flapping angle expressed as a Fourier series up to the second harmonic:

$$\mathbf{b} = \mathbf{b}_0 + \mathbf{b}_{1c} \cos \mathbf{y} + \mathbf{b}_{1s} \sin \mathbf{y} + \mathbf{b}_{2c} \cos 2\mathbf{y} + \mathbf{b}_{2s} \sin 2\mathbf{y} \quad (15)$$

Substituting (12), (14) and (15) into (13) gives the angle of attack when the second harmonic input is applied:

$$\begin{aligned} \mathbf{a}_{2H} &= \Delta \mathbf{q}_0 + \mathbf{q}_{1s} \sin \mathbf{y} + \mathbf{q}_{2c} \cos 2\mathbf{y} + \mathbf{q}_{2s} \sin 2\mathbf{y} + \frac{1}{(x + \mathbf{m} \sin \mathbf{y})} \times \\ &\left\{ \left[ \mathbf{1} - (\mathbf{b}_0 + \mathbf{b}_{1c} \cos \mathbf{y} + \mathbf{b}_{1s} \sin \mathbf{y} + \mathbf{b}_{2c} \cos 2\mathbf{y} + \mathbf{b}_{2s} \sin 2\mathbf{y}) \cdot \right. \right. \\ &\left. \left. \mathbf{m} \cos \mathbf{y} \right] + x (\mathbf{b}_{1c} \sin \mathbf{y} - \mathbf{b}_{1s} \cos \mathbf{y} + 2\mathbf{b}_{2c} \sin 2\mathbf{y} - 2\mathbf{b}_{2s} \cos 2\mathbf{y}) \right\} \\ &= \mathbf{a}_{1H} + \Delta \mathbf{a}_{2H} \end{aligned} \quad (16)$$

$\alpha_{2H}$  and is a summation of the incidence from the first harmonic  $\alpha_{1H}$  plus the change in the incidence distribution due to the application of the second harmonic control  $\Delta\alpha_{2H}$ . As pointed out above, one has to ensure that  $\Delta\alpha_{2H}$  is minimum in the region Area 2 between 225 and 315 deg in order to make sure that  $\alpha_{2H}$  decreases in this region. Looking at (16), the change in incidence distribution due to the application of the second harmonic control is given by:

$$\Delta\mathbf{a}_{2H} = \Delta\mathbf{q}_0 + \mathbf{q}_{2c} \cos 2\mathbf{y} + \mathbf{q}_{2s} \sin 2\mathbf{y} - \frac{(\mathbf{b}_{2c} \cos 2\mathbf{y} + \mathbf{b}_{2s} \sin 2\mathbf{y})\mathbf{m} \cos \mathbf{y}}{(x + \mathbf{m} \sin \mathbf{y})} + \frac{(2\mathbf{b}_{2c} \sin 2\mathbf{y} - 2\mathbf{b}_{2s} \cos 2\mathbf{y})x}{(x + \mathbf{m} \sin \mathbf{y})} \quad (17)$$

The value  $\Delta\theta_0$  was also included as it originates from the second harmonic input. The value of  $\mu$  does not have a serious effect in equation (17) – except near the blade root – and thus can be neglected. The variation in incidence becomes then:

$$\Delta\alpha_{2H} \cong \mathbf{q}_0 + \mathbf{q}_{2c} \cos 2\mathbf{y} + \mathbf{q}_{2s} \sin 2\mathbf{y} + 2\mathbf{b}_{2c} \sin 2\mathbf{y} - 2\mathbf{b}_{2s} \cos 2\mathbf{y} \quad (18)$$

Reference 8 derived approximations for the coefficients  $\mathbf{b}_{2c}$  and  $\mathbf{b}_{2s}$  as a function of the higher harmonic input in the form:

$$\mathbf{b}_{2c} \cong \frac{\mathbf{q}_{2c} / 2 + \mathbf{g} / 24 \cdot \mathbf{q}_{2s}}{12 / \mathbf{g} + \mathbf{g} / 12} \quad (19)$$

$$\mathbf{b}_{2s} \cong \frac{-\mathbf{g} / 24 \cdot \mathbf{q}_{2c} + \mathbf{q}_{2s} / 2}{12 / \mathbf{g} - \mathbf{g} / 12}$$

where  $\gamma$  is the Lock number ( $\gamma = 9.35$  for the FGR model).

Bearing in mind the expressions for the  $\Delta\theta_0$ ,  $\theta_{2c}$  and  $\theta_{2s}$ , it follows that  $\Delta\alpha_{2H}$  is a periodic function of azimuth  $\psi$  depending on the amplitude of the second harmonic amplitude  $C$ , the phase  $\phi$ , and Lock number  $\gamma$ . The azimuth angles  $\psi_i$  where  $\Delta\alpha_{2H}$  achieve a minimum during a complete cycle can be deduced by imposing:

$$\frac{\partial(\Delta\mathbf{a}_{2H})}{\partial\mathbf{y}} = 0 \rightarrow \mathbf{y}_i \quad \frac{\partial^2(\Delta\mathbf{a}_{2H})}{\partial^2\mathbf{y}}(\mathbf{y}_i) > 0 \quad (20)$$

Solving the first equation (20) gives:

$$\mathbf{y}_i = \frac{1}{2} \arctan\left(\frac{\mathbf{q}_{2s} + 2\mathbf{b}_{2c}}{\mathbf{q}_{2c} - 2\mathbf{b}_{2s}}\right) + \frac{n\pi}{2}, \quad n \in \mathbb{Z} \quad (21)$$

The sign of the second derivative  $\frac{\partial^2(\Delta\mathbf{a}_{2H})}{\partial^2\mathbf{y}}(\mathbf{y}_i)$  was determined for the FGR model and represented as shown in Figure 9 as a function of the phase angle,  $\phi$ , for different amplitudes of the higher harmonic.

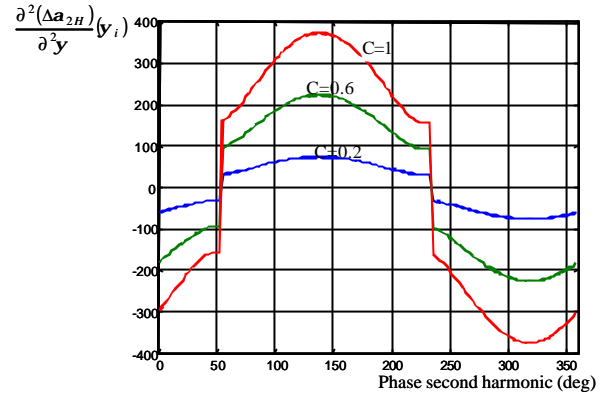


Figure 9: Second derivative of the angle of attack  $\Delta\mathbf{a}_{2H}$

Looking at this figure and bearing in mind that we are investigating the minimum of the function  $\Delta\alpha_{2H}$ , it follows that the phase angle  $\phi$  to be investigated varies between 55-232 deg. Furthermore, the points  $\psi_i$  of interest are situated in Area 2 between 225 and 315 deg. Figure 10 presents the variation of the azimuth  $\psi_I$  corresponding to a minimum/maximum  $\Delta\alpha_{2H}$  phased w.r.t. the initial control. Intersecting the two regions of interest gives the phase angle corresponding to a reduction in the angle-of-attack after the application of the second harmonic. For example, to achieve a minimum in  $\Delta\alpha_{2H}$  at azimuth 270 deg one has to apply the second harmonic phased 170 deg back to the pulse application.

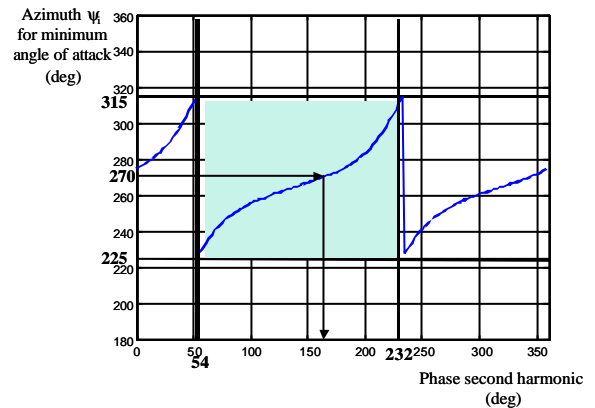


Figure 10: Deducing the phase angle corresponding to a minimum in incidence variation in Area 2 of the disc

It can be seen that the variation in amplitude of the second harmonic did not prove to be as important for the shifting of the retreating blade stall. However, the angle of attack  $\alpha_{2H}$  after the application of the second harmonic depends also on the amplitude of this higher harmonic and will help to minimize the total  $\alpha_{2H}$  on the retreating side Area 2.

Consider now the superposition of the second harmonic control onto a pulse input of 1 second applied when the helicopter is flying at 150 kn. We represent the stall patterns in cycle 6 after the pulse was applied for different variations in the phase if the higher harmonic phase  $\phi=[0\ 45\ 90\ 145\ 180\ 225\ 270\ 315]$  deg. The input amplitude was 0.2 in (approximate 0.6 deg). Figure 11 summarises the results given by the FGR model.

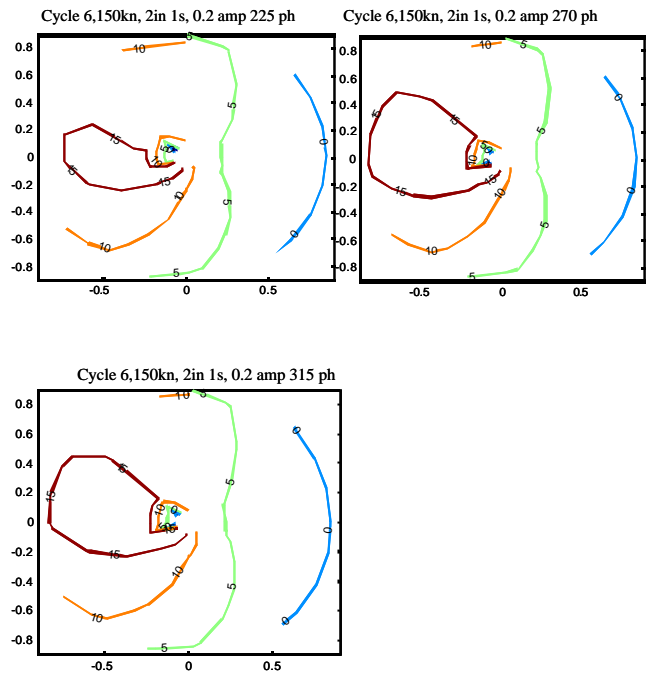
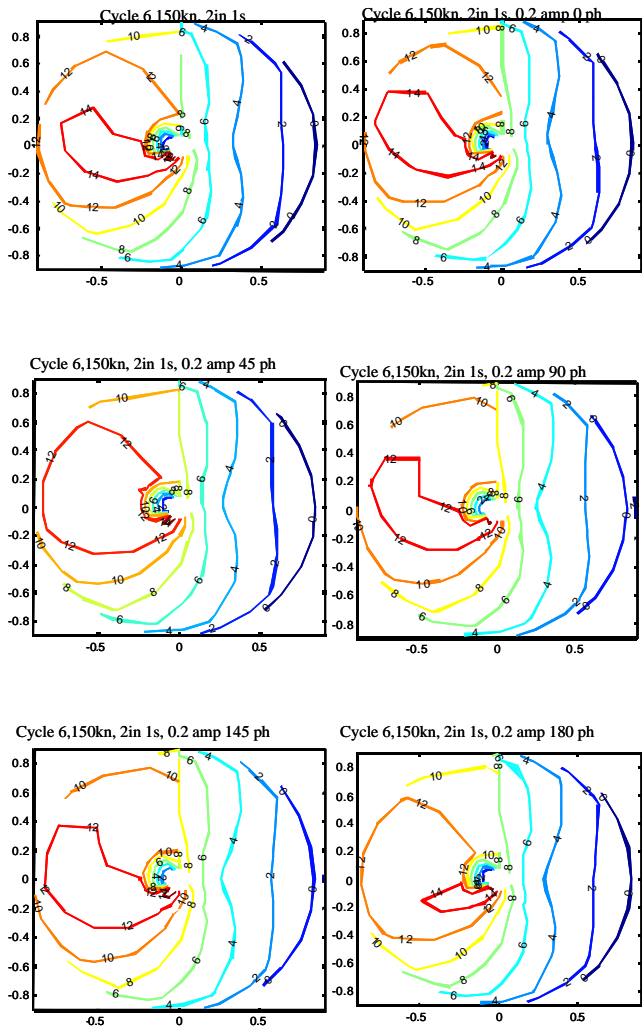
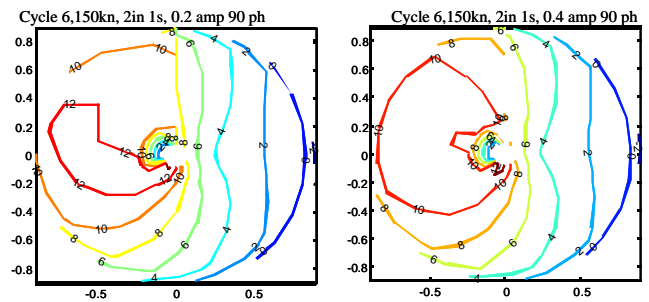


Figure 11: Effect of variation of second harmonic phase on the incidence distribution

From this set of figures it may be concluded that a reduction in the angle of attack distribution on the retreating blade is obtained for phase angles of 45, 90 and 180 deg. Retreating blade stall actually increases when superimposing a second harmonic input phased 225, 270 and 315 deg relative to the input.

Once the optimum phase was determined, the amplitude of the second harmonic was varied. The results can be seen in Figure 12.



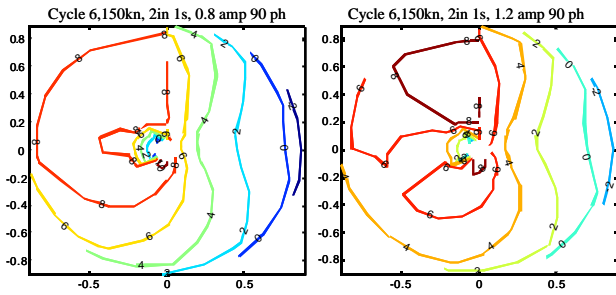


Figure 12: Effect of variation of second harmonic amplitude on the incidence distribution

From Figure 12 one may see that, for an appropriately phased second harmonic input, increasing the amplitude of the higher harmonic reduces the region of the rotor disc where the blade stalls.

Let us return now to the performance and loading issues discussed earlier in the paper and determine how the application of the open-loop second harmonic input affects the agility and load quickness parameters.

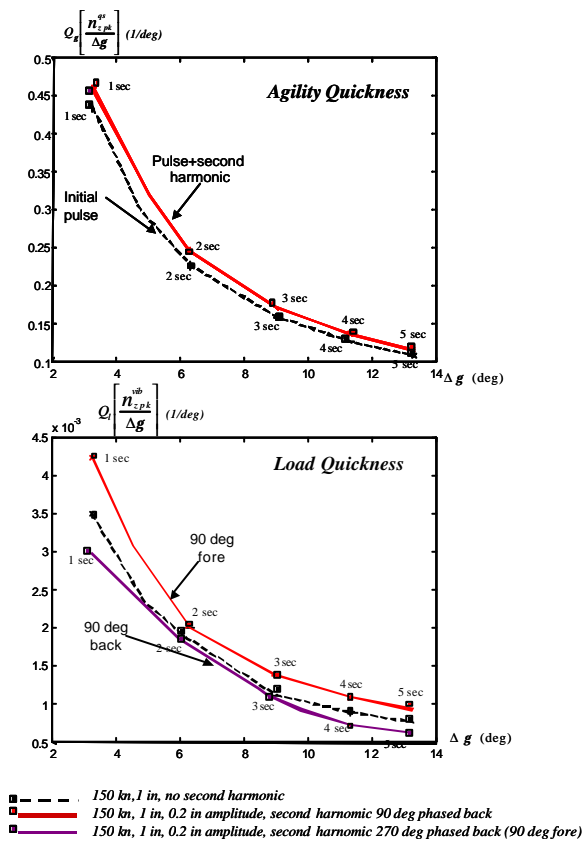


Figure 13: Agility and load quickness charts of second harmonic inputs superimposed on the 1 in pull-up manoeuvres

First, a second harmonic pulse was superimposed onto a 1 in initial pulse. Figure 13 presents the new charts for the case of a second harmonic input of 0.2 in amplitude, 90 deg phased back, and a second harmonic of 0.2 in amplitude, 270 deg phased back (which is equivalent to 90 deg phased forward of course). In both cases, the flight path quickness indicates an increase of the helicopter agility (this is mainly due to the amplitude of the second harmonic). However, while the 90 deg phased input reduces the vibratory activity, the 270 deg phased input actually increases the vibratory load activity on the helicopter. Thus, the former case, where the second harmonic control has a 90 degree phase relative to the input can be used for both structural alleviation and helicopter performance improvement. Figure 13 indicates a decreasing up to 20% in the load quickness parameter when 90 deg back of course second harmonic input is used.

Figure 14 presents the agility and load quickness charts, this time superimposing second harmonics on a 2 in initial pulse.

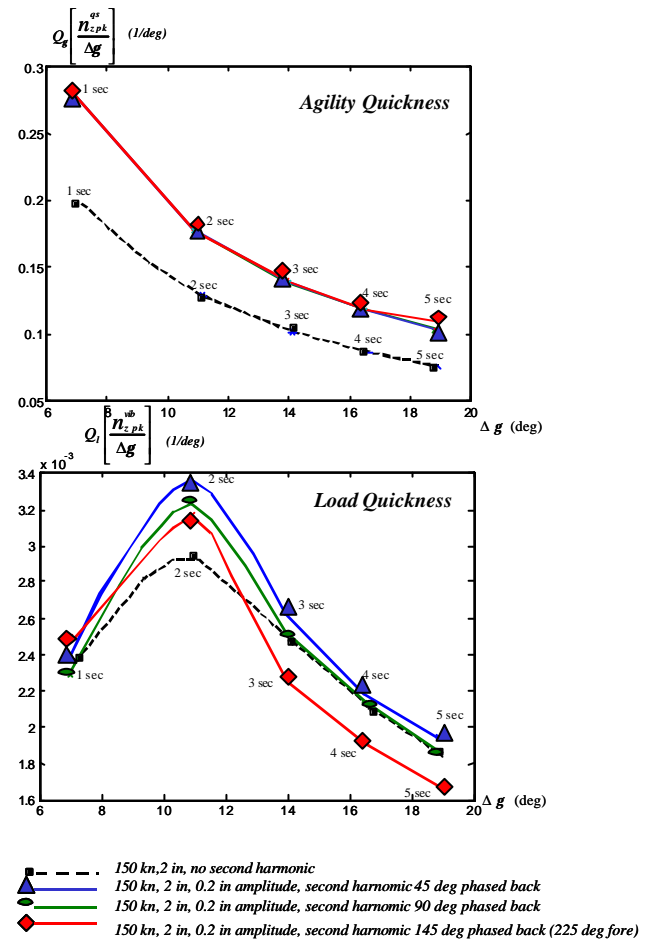


Figure 14: Agility and load quickness charts obtained varying the phase of the second harmonic superimposed on 2 in pull-up manoeuvres

- - - - 150 kn, 1 in, no second harmonic
- - - - 150 kn, 1 in, 0.2 in amplitude, second harmonic 90 deg phased back
- - - - 150 kn, 1 in, 0.2 in amplitude, second harmonic 270 deg phased back (90 deg fore)

- - - - 150 kn, 2 in, no second harmonic
- ▲ - - - 150 kn, 2 in, 0.2 in amplitude, second harmonic 45 deg phased back
- - - - 150 kn, 2 in, 0.2 in amplitude, second harmonic 90 deg phased back
- ◆ - - - 150 kn, 2 in, 0.2 in amplitude, second harmonic 145 deg phased back (225 deg fore)

In these cases, it can be seen that although a 45 and a 90 deg phased second harmonic input decreases the blade incidence on the retreating side of the disc, the load quickness increases, especially for short pulse durations. A phase of 145 deg is more suitable for structural alleviation, resulting in an increase of helicopter performance and decrease of retreating disc stalled area.

Figure 15 shows the agility and load quickness diagrams when the amplitude of the second harmonic input was varied, keeping a constant phase of 90 deg. From this figure it may be concluded that increasing the amplitude of the second harmonic by too much can result in an increase in the load quickness, although the retreating blade stall patterns actually reduce (see Figure 12).

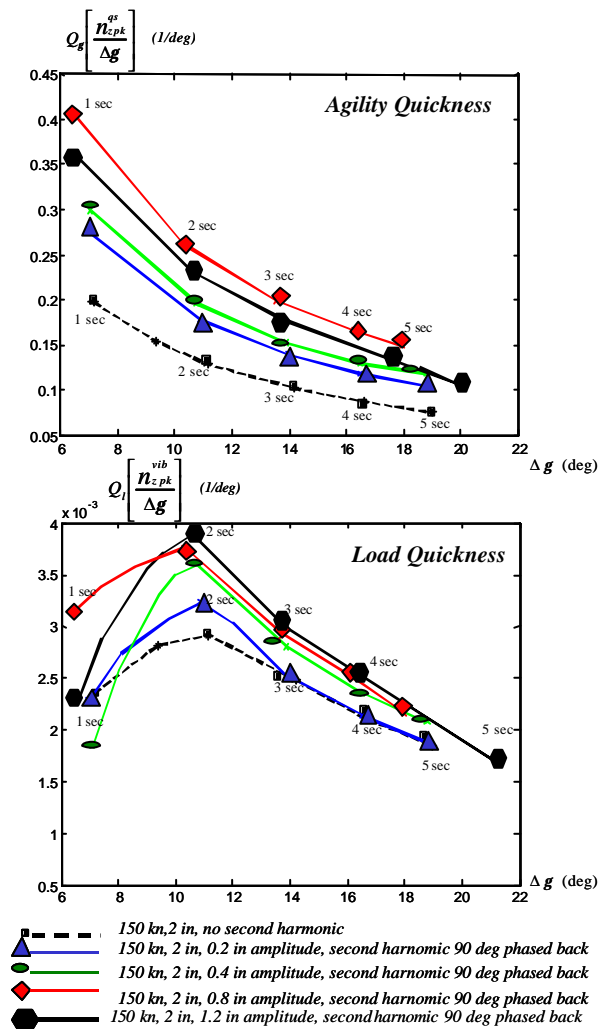


Figure 15: Agility and load quickness charts obtained varying the amplitude of the second harmonic superimposed on 2 in pull-up manoeuvres

## CONCLUSIONS AND RECOMMENDATIONS

The present paper has presented a new approach to multi-disciplinary optimisation and the assessment of handling qualities utilising ADS-33 metrics. The paper has concentrated on assessing the handling qualities in vertical manoeuvres in the pitch axis in forward flight. In this context, the ADS-33 low-speed attitude quickness parameter was developed into two new metrics:

- 1) *agility quickness (gamma quickness)*, defined as the ratio of peak quasi-steady normal acceleration to flight path angle change during vertical manoeuvres, and,
- 2) *vibratory load quickness*, defined as the ratio of peak amplitude of vibratory load component to flight path angle change. For manoeuvres in the vertical axis, the peak amplitude of vibratory load can be related to the peak amplitude of the 4/rev component of the hub vertical shear.

While the agility quickness characterises the helicopter performance during vertical manoeuvring flight, the load quickness is used to quantify the build up of loads in the rotor. For small-amplitude inputs, agility quickness is mainly represented by the helicopter's heave bandwidth; for large-amplitude inputs, agility quickness depends mainly on the quasi-steady pitch rate and reduces to the attitude quickness.

These new metrics were used to optimise the rotor loading without compromising the aircraft manoeuvrability. In this context, open-loop second harmonic cyclic control inputs were superimposed onto the primary longitudinal cyclic pitch control in a pull-up manoeuvre. The primary case investigated was a rather unusual behaviour of the FLIGHTLAB FGR model in the Liverpool simulator when executing pull-up manoeuvres from high speed flight conditions (120 and 150 kn). The pilots observed that, when flying such manoeuvres, the engine torque increased rather than decreased as expected. Further analysis of the results revealed that this behaviour was a consequence of the rotor stalling on the retreating side. In principle, modifying the rotor disc stall patterns would result in improvement of both performance and vibratory activity. The method chosen to modify the stall patterns consisted of the application of open-loop second harmonic control inputs superimposed on the primary cyclic pitch control. It was demonstrated that for FGR model, the second harmonic feathering control has to be phased back w.r.t. the first harmonic feathering control with a phase angle situated between 90 and 145 degree. Such a phase results in shifting the retreating blade stall patterns around the rotor disc azimuth. The pull-up manoeuvres at high velocities were flown again, this time applying a first + second harmonic 'properly' phased longitudinal cyclic input and the corresponding agility and load quickness charts were re-plotted. It was demonstrated that an application of the

second harmonic pitch control input resulted in an increase in the agility quickness parameter and thus of the helicopter performance, irrespective of the magnitude and phase of the applied second harmonic input. Varying the amplitude of the second harmonic indicated that a threshold exists above which there is a reduction in the load quickness parameter, although the blade incidence on the retreating side of the disc is still reduced. Concerning the load vibratory quickness, application of a second harmonic input, 'properly' phased relative to the first harmonic pitch control, is fundamental for load alleviation. When correctly phased, the rotor loading can be optimised without compromising the aircraft manoeuvrability. In this way it was demonstrated how the new metrics introduced in the present paper can be used to gain insightful multi-disciplinary optimisation.

As noted in the Introduction, the work presented here represents a first step in a collaboration between Delft and Liverpool in the area of handling qualities and multi-disciplinary optimisation. Several aspects of the work will be extended as the collaboration continues. Agility/HQ boundaries need to be developed for different mission task elements and applied to agility quickness. Likewise, goals for load alleviation need to be postulated. The use of feedback in the application of the higher harmonic control is an obvious candidate for developing improved performance. It is also considered that the application of a combination of 2<sup>nd</sup> + 3<sup>rd</sup> harmonic feathering control may result in further improvement of structural loads distribution. In the future these quickness metrics, enabling an inter-disciplinary approach to rotorcraft design, will be extended to a larger database of situations. In particular the higher fidelity rotor structural and aerodynamic representations available in FLIGHTLAB will be used to obtain more realistic solutions.

#### ACKNOWLEDGEMENTS

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