

# Analysis of flow development in centrifugal atomization: Part I. Film thickness of a fully spreading melt

Y Y Zhao

Department of Engineering, The University of Liverpool, Brownlow Hill, Liverpool L69 3GH, UK

Received 11 November 2003

Published 9 August 2004

Online at [stacks.iop.org/MSMSE/12/959](http://stacks.iop.org/MSMSE/12/959)

doi:10.1088/0965-0393/12/5/013

## Abstract

Centrifugal atomization of metal melts is a cost-effective process for powder production and spray deposition. The properties of the as-produced powder and deposit are determined primarily by the characteristics of the atomized droplets, which in turn are largely dependent on the flow development of the melt on the atomizer. This paper develops a model for analysing the flow development of a fully spreading melt on and off the atomizing cup. The model can be used to calculate the velocity and film thickness of the melt as a function of melt volume flow rate, cup rotation speed, cup radius and cup slope angle, as well as to predict the trajectory of the spray off the cup. The model implies that the disintegration of a fully spreading melt takes place in the region just off the cup edge and the film thickness at the cup edge is a critical factor determining the sizes of the resultant droplets. The film thickness at the cup edge is shown to decrease with decreasing volume flow rate, with increasing cup rotation speed, with increasing cup radius and with decreasing cup slope angle.

## Nomenclature

$c_1$	a compound parameter
$c_2$	a compound parameter
$d\vec{F}$	centrifugal force (vector)
$dA$	area of the top surface of a small volume of melt
$dA_s$	area of the surface of a small volume of melt perpendicular to the slope direction
$dF_h$	normal component of centrifugal force to the cup wall
$dF_r$	radial component of centrifugal force
$dF_s$	slope-direction component of centrifugal force

$dF_\theta$	azimuthal component of centrifugal force
$dV$	volume of a small element of the melt
$H$	film thickness of the melt on the cup
$H_d$	film thickness of the melt off the cup
$h$	normal distance from the cup wall surface
$Q$	melt volume flow rate
$\vec{r}$	radial coordinate (vector)
$r$	radial coordinate in the cylindrical system
$r_c$	hydraulic jump radius
$r_o$	cup radius
$t$	time
$\vec{v}$	melt velocity relative to the cup (vector)
$\bar{v}_s$	mean slope-direction melt velocity
$\bar{v}_\theta$	mean azimuthal melt velocity relative to the cup
$v_d$	divergence melt velocity off the cup
$v_o$	slope-direction melt velocity at the cup edge
$v_r$	radial melt velocity off the cup
$v_s$	slope-direction melt velocity
$v_t$	absolute tangential melt velocity at the cup edge
$v_\theta$	azimuthal melt velocity relative to the cup
$x$	coordinate in Cartesian system
$y$	coordinate in Cartesian system
$z$	coordinate in Cartesian system
$\alpha$	cup wall slope angle
$\beta$	inclination angle of melt film off the cup
$\phi$	degree of slippage
$\kappa$	ratio between the slope-direction and tangential velocities
$\mu$	viscosity of the melt
$\rho$	specific density of the melt
$\tau_s$	slope-direction viscous stress
$\tau_\theta$	azimuthal viscous stress
$\Omega$	cup rotation speed (rps)
$\vec{\omega}$	cup rotation speed (vector)
$\omega$	cup rotation speed (radian $s^{-1}$ )

## 1. Introduction

Centrifugal atomization of water-based liquids and slurries has been a well-established process in the chemical, agricultural and food industries for several decades [1, 2]. More recently, centrifugal atomization has been developed for manufacturing powders or ring-shaped deposits from a range of metallic materials, including Sn, Pb, Al, Zn, Mg, Ti, Ni, Co and their alloys [3–12]. The process utilizes the centrifugal force provided by a rapidly rotating atomizer, either a flat disc or a cup, to break up a liquid stream into a spray of droplets that either solidify in flight to form high quality powders or deposit onto a substrate to form microstructurally refined and chemically homogeneous preforms. The main advantage of centrifugal atomization over gas atomization is its high energy efficiency and therefore low production cost [7]. Centrifugal

atomization has the highest efficiency among the commercial atomization techniques because nearly all the rotational work of the atomizer is consumed in directly breaking up the liquid and accelerating the droplets [7]. There is no consumption of gases in the process except in the case of maintaining a protective atmosphere in the processing chamber when an inert environment is required. Another advantage of centrifugal atomization is its capability of producing spherical powder particles with a controlled narrow size distribution, which is often desirable for many applications.

Despite the aforementioned advantages, centrifugal atomization is not as widely applied on an industrial scale for powder production and spray deposition as one might expect. This is mainly due to a few technical limitations. First, it is difficult to design and fabricate an atomizer combining efficient cooling and high rotational speeds. Second, a large process chamber is often needed for the atomized droplets to solidify completely in flight in powder production. Third, the preforms produced by centrifugal spray deposition are at present limited to cylindrical rings. Finally, the droplet sizes tend to be coarser than required for many applications. The realization of the full potential of centrifugal atomization for industrial applications is also prohibited by the lack of in-depth scientific understanding of the process.

In centrifugal atomization for powder production or spray deposition, the powder characteristics, deposit microstructures and the associated mechanical properties are determined primarily by the size, temperature and velocity distribution of the atomized droplets. The atomization process in turn is largely dependent on the flow development of the melt, i.e. molten metal, on the atomizer.

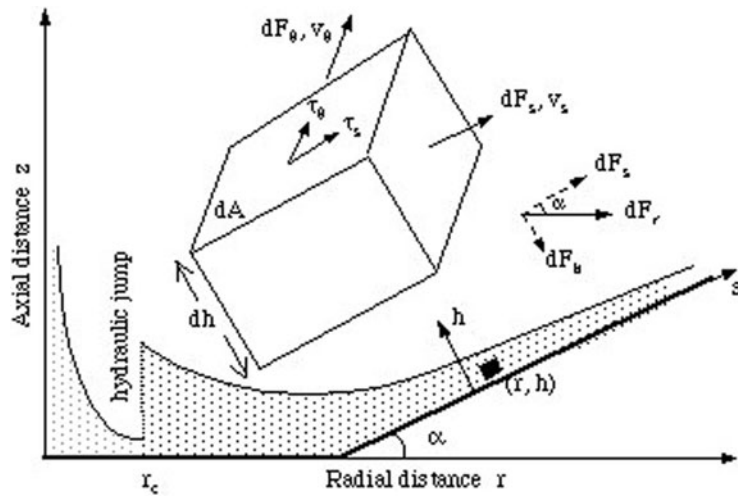
The interaction between the melt and the atomizer is complex. After the melt stream impinges on the atomizer, the centrifugal force accelerates the melt to spread outwards to the edge of the atomizer at high tangential and radial velocities. At a high rotation speed, a hydraulic jump often occurs on the atomizer. The hydraulic jump is an annular discontinuity in the flow manifested by an abrupt increase in the melt thickness and correspondingly a reduction in the radial velocity and, for a flat disc, occurs at a radius approximated by [13]

$$r_c = 0.55 \left( \frac{\rho Q^2}{\mu \omega} \right)^{1/4}, \quad (1)$$

where  $\rho$  is the specific density,  $\mu$  the viscosity,  $Q$  the volume flow rate of the melt and  $\omega$  is the rotation speed of the disc (scalar, in radian  $s^{-1}$ ).

From a practical point of view, the region of interest is the flow directly prior to the disintegration of the melt. In most cases, it is sufficient to only consider the flow development after the hydraulic jump, where the flow is completely controlled by the rotating atomizer and is independent of the initial velocity of the melt stream at the impingement on the atomizer. For an ideal melt that spreads fully on the atomizer, the flow would be a continuous thin film covering the surface of the atomizer. Analytical and simplified numerical models have been developed to calculate the film thickness and the radial and tangential velocity distributions of the melt on a flat disc [1, 14, 15]. These models have provided a foundation for understanding how the processing parameters, such as disc radius, rotation speed and melt flow rate, would affect the film thickness and the subsequently formed droplets. These models also make quantitative correlation between the processing conditions and the droplet size distribution possible. However, the development of the melt flow on and beyond a rotating cup, which is more widely used in practice, has not been analysed quantitatively.

This paper is to develop a model for the flow development of a fully spreading melt on and immediately off a rotating cup with a sloped wall, adopting a similar approach as used in developing the analytical model for a flat disc [14]. The model will be able to predict the film thickness of the melt in the vicinity of the cup edge as a function of the physical properties



**Figure 1.** Schematic diagram of the melt film on a rotating cup as well as the centrifugal forces and viscous stresses exerted on a small volume.

of the melt and the operating conditions in centrifugal atomization. This paper also discusses how the processing parameters would affect the droplet size distribution.

**2. Model**

*2.1. Flow on cup*

The present model is concerned with the development of the melt flow on the sloped wall of a rotating cup after a hydraulic jump. The assumptions made in developing the model are (1) the melt flow is continuous, axi-symmetric and steady; (2) the inertial and gravitational forces are negligible compared with the centrifugal force; (3) the melt is a Newtonian fluid with a constant viscosity and (4) the melt fully spreads on the cup and has no relative movement at the melt–cup interface.

Figure 1 shows schematically the thickness profile of the melt on a rotating cup and the centrifugal forces exerted on a small volume of the melt, as well as the viscous stresses and velocities in the directions of interest. Let us consider an infinitesimally small volume of the melt in the thin film on the sloped wall of the cup as shown in figure 1. The centrifugal force exerted on a small melt with a volume of  $dV$  is given by [16]

$$d\vec{F} = -\rho[2\vec{\omega} \times \vec{v} + \vec{\omega} \times (\vec{\omega} \times \vec{r})] dV, \tag{2}$$

where  $\vec{\omega}$  is the rotation speed of the cup (vector),  $\vec{v}$  the velocity of the liquid relative to the rotating cup (vector) and  $\vec{r}$  is the radial coordinate (vector).

As the melt flow on the cup wall virtually has only two velocity components, one in the azimuthal direction and one in the slope direction ( $s$  direction in figure 1), a convenient approach is to consider the components of the centrifugal force in these two directions. Let us first consider the centrifugal force and the resultant viscous stress and velocity in the slope direction. Assuming that the melt does not slip very much on the cup along the azimuthal direction such that the azimuthal velocity of the liquid relative to the rotating cup is much smaller than the local azimuthal velocity of the cup, the first term on the right-hand side of

equation (2) can be neglected in determining the radial centrifugal force. The radial component of the centrifugal force can therefore be approximated by

$$dF_r \approx \rho\omega^2 r dV, \quad (3)$$

where  $r$  is the radial coordinate of the small volume of melt in the cylindrical coordinate system (scalar).

The radial component of the centrifugal force,  $dF_r$ , can be further separated into a component in the slope direction,  $dF_s$ , and a component in the normal direction to the cup wall,  $dF_h$  ( $dF_h$  has no direct effect on the flow development in both the azimuthal and slope directions), as shown in figure 1. With a negligible inertial force, the centrifugal force in the slope direction is balanced by the viscous forces exerted on the two planes parallel to the cup wall. The difference in the viscous forces on the two planes is thus given by

$$d\tau_s = \frac{dF_s}{dA} = \frac{\cos \alpha dF_r}{dA} \approx \rho\omega^2 r \cos \alpha dh, \quad (4)$$

where  $\alpha$  is the slope angle of the cup wall, i.e. the angle between the cup wall and the radial direction,  $dA$  the area of the top or bottom surface (in parallel to the cup wall) of the small volume in consideration and  $dh$  is the thickness of the small volume.

Because the viscous stress at the free top surface of the melt film is zero, integrating equation (4) gives the viscous stress in the slope plane at any normal position to the cup wall,  $h$ , as

$$\tau_s = \rho\omega^2 r \cos \alpha (H - h), \quad (5)$$

where  $H$  is the thickness of the melt film at a radial distance of  $r$ . For a Newtonian fluid, the viscous stress in the slope plane is related to the gradient of the slope-direction liquid velocity,  $v_s$ , in the normal direction and is given by

$$\tau_s = \mu \frac{\partial v_s}{\partial h}. \quad (6)$$

Because the melt has a zero velocity relative to the cup at the melt–cup interface, combining equations (5) and (6) and then integrating the combined equation gives the slope-direction velocity at any normal position  $h$  ( $0 \leq h \leq H$ ) as

$$v_s = \frac{\rho\omega^2 r \cos \alpha}{\mu} \left[ Hh - \frac{1}{2}h^2 \right]. \quad (7)$$

For a steady-state incompressible flow, the volume flow rate of the melt across the cylindrical plane at any radius  $r$  must be equal to the initial volume flow rate of the melt stream flowing onto the cup. To satisfy the volume conservation criterion, the slope-direction velocity profile in the film along the normal direction to the cup wall needs to obey the following equation:

$$\int_0^H 2\pi r v_s dh = Q. \quad (8)$$

Substituting  $v_s$  in equation (7) into equation (8) and integrating the left-hand side gives the thickness of the melt film at a radius  $r$  as

$$H = \left[ \frac{3\mu Q}{2\pi\rho\omega^2 r^2 \cos \alpha} \right]^{1/3}. \quad (9)$$

The mean slope-direction velocity of the melt at a radius  $r$  is therefore

$$\bar{v}_s = \frac{Q}{2\pi r H} = \frac{1}{2} \left[ \frac{2\rho\omega^2 Q^2 \cos \alpha}{3\pi^2 \mu r} \right]^{1/3}. \quad (10)$$

Let us now consider the centrifugal force and the resultant viscous stress and velocity in the azimuthal direction. From equation (2), the azimuthal component of the centrifugal force exerted on the small volume of melt is

$$dF_\theta = -2\rho\omega v_s \cos \alpha dV. \quad (11)$$

Following the same approach as used in the derivation of the slope-direction velocity, the azimuthal viscous stress in the melt,  $\tau_\theta$ , and the azimuthal velocity of the melt relative to the cup,  $v_\theta$ , at any normal position  $h$  ( $0 \leq h \leq H$ ) can be derived as

$$\begin{aligned} \tau_\theta &= \frac{\rho^2 \omega^3 r \cos^2 \alpha}{3\mu} [2H^3 - 3Hh^2 + h^3], \\ v_\theta &= -\frac{\rho^2 \omega^3 r \cos^2 \alpha}{12\mu^2} [8H^3 h - 4Hh^3 + h^4]. \end{aligned} \quad (12)$$

The mean azimuthal velocity of the melt relative to the cup is therefore

$$\bar{v}_\theta = \frac{\int_0^H v_\theta dh}{H} = -\omega r \left\{ \frac{3}{5} \left[ \frac{2\rho Q^2 \cos \alpha}{3\pi^2 \mu \omega r^4} \right]^{2/3} \right\}. \quad (13)$$

The azimuthal velocity of the melt relative to the cup is negative because there is slippage in the azimuthal direction between the melt and cup. The degree of slippage at a radius  $r$  can be expressed by the ratio between the relative azimuthal velocity of the melt and the azimuthal velocity of the cup at  $r$  and is given by

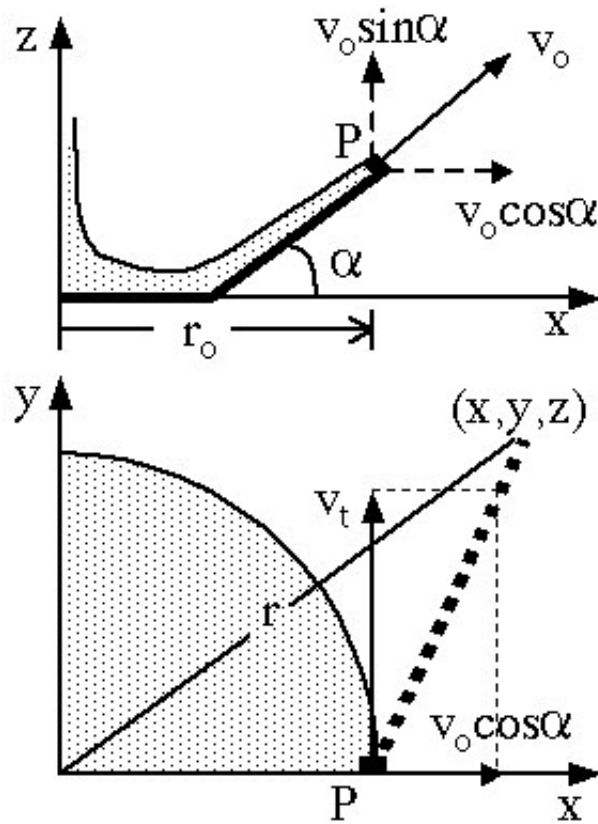
$$\phi = \frac{|\bar{v}_\theta|}{\omega r} = \frac{3}{5} \left[ \frac{2\rho Q^2 \cos \alpha}{3\pi^2 \mu \omega r^4} \right]^{2/3}. \quad (14)$$

## 2.2. Flow off cup

When the melt is ejected from the edge of the rotating cup, the flow will no longer be controlled by the rotating cup. However, the flow off the cup is largely determined by the direction and magnitude of the velocity of the melt at the point of leaving the cup. The melt film at the periphery of the rotating cup is usually very thin and can therefore be considered to have a uniform velocity across the film thickness in examining the flow beyond the rotating cup. Because the flow is axi-symmetric, we need only to consider a small element of melt at the periphery of the cup and its flight trajectory. Let us consider a small volume at point P at the periphery of a cup with a radius of  $r_o$ , as shown schematically in figure 2. This small element has a velocity with one component in the slope direction and one in the tangential direction. According to equations (10) and (13), the slope-direction velocity,  $v_o$ , and the absolute tangential velocity,  $v_t$ , of the element at point P can be calculated as

$$\begin{aligned} v_o &= \frac{1}{2} \left[ \frac{2\rho\omega^2 Q^2 \cos \alpha}{3\pi^2 \mu r_o} \right]^{1/3}, \\ v_t &= \omega r \left\{ 1 - \frac{3}{5} \left[ \frac{2\rho Q^2 \cos \alpha}{3\pi^2 \mu \omega r_o^4} \right]^{2/3} \right\}. \end{aligned} \quad (15)$$

As far the atomization of the melt is concerned, the regime of main interest is the melt flow within a small distance from the cup periphery. In this region, the effect of gravity and the drag force exerted by the air on the flow are small and can be neglected. The magnitude and direction of the velocity of the element can be regarded as constant throughout its flight in the air. The velocity components of the element in the  $x$ ,  $y$  and  $z$  directions in the Cartesian coordinate



**Figure 2.** Schematic diagram showing the velocity components of a small volume of melt at the edge of the atomizing cup and the top view of the travel trajectory of this small volume after having left the cup.

system are  $v_0 \cos \alpha$ ,  $v_t$  and  $v_0 \sin \alpha$ , respectively, as shown in figure 2. At a time  $t$  after the element has been ejected from the cup, the coordinates of the element ( $x$ ,  $y$ ,  $z$ ) would be

$$x = r_0 + v_0 \cos \alpha \cdot t, \quad y = v_t t, \quad z = v_0 \sin \alpha \cdot t. \quad (16)$$

The radial distance of the element from the centre of the cup is therefore varying with time  $t$  and is expressed by

$$r = \sqrt{x^2 + y^2} = \sqrt{(r_0 + v_0 \cos \alpha \cdot t)^2 + v_t^2 t^2}. \quad (17)$$

Because the melt film ejected from the cup is an axi-symmetric flow, the thickness of the film is independent of the azimuthal velocity and is determined by a divergent velocity composed of the radial and axial velocity components. As a consequence, only the radial and axial velocities of a representative element need to be considered in examining the changes in the film thickness. Differentiating equation (17) with respect to  $t$  gives the radial velocity of the film as a function of radial distance as

$$v_r = \frac{dr}{dt} = \sqrt{(v_0 \cos \alpha)^2 + v_t^2 \left(1 - \frac{r_0^2}{r^2}\right)}. \quad (18)$$

The magnitude of the divergence velocity is thus

$$\begin{aligned} v_d &= \sqrt{v_r^2 + (v_o \sin \alpha)^2} = \sqrt{v_o^2 + v_t^2 \left(1 - \frac{r_o^2}{r^2}\right)} \\ &\approx \sqrt{v_o^2 + v_t^2} \quad (r \gg r_o). \end{aligned} \quad (19)$$

It should be noted that, although the velocity of a small element of the melt has a constant direction and magnitude throughout its flight after being ejected from the cup, the direction and magnitude of the divergence velocity of the film as a whole vary with radial distance off the cup. To satisfy the volume conservation criterion, the film thickness must change according to the magnitude of the divergence velocity. If the melt remains a continuous film without disintegration, the thickness of the film off the cup is simply

$$H_d = \frac{Q}{2\pi r v_d} = \frac{Q}{2\pi \sqrt{(v_o^2 + v_t^2)r^2 - v_t^2 r_o^2}}. \quad (20)$$

### 3. Illustrations and discussions

The current model is based on the assumption that the centrifugal force dominates the liquid flow and the inertial and gravitational forces are negligible. As the flow on an atomizing disc or cup is largely planar and we are only concerned with the radial and azimuthal velocities, neglecting the gravitational force should not result in significant errors. The omission of the inertial force, however, deserves some consideration.

The liquid flow on the atomizing disc or cup is subject to an inertial force in the radial or slope direction, respectively. The inertial force on a volume of melt is associated with the change in the momentum between the two surfaces of the volume in the radial or slope direction. The inertial force exerted on a small volume as shown in figure 1,  $dV$ , can be expressed as

$$dF_i = \rho \bar{v}_s dA_s d\bar{v}_s, \quad (21)$$

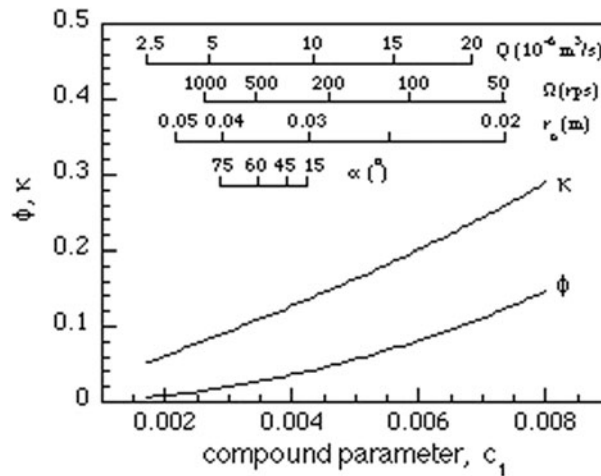
where  $dA_s$  is the area of the surface perpendicular to the slope direction. Given the relevant terms in equations (3), (4) and (10), the ratio between the inertial force and the centrifugal force in the slope direction can be determined as

$$\frac{dF_i}{dF_s} = \frac{\bar{v}_s}{\omega^2 r} \left| \frac{d\bar{v}_s}{dr} \right| = \frac{1}{12} \left( \frac{2\rho Q^2 \cos \alpha}{3\pi^2 \mu \omega} \right)^{2/3} r^{-(8/3)}. \quad (22)$$

For the centrifugal atomization conditions normally used in practice, the ratio between the inertial and centrifugal forces is less than 0.05. The current analytical model is therefore an acceptable simplification. For a wider range of process conditions where the inertia force is not negligible, a comprehensive approach based on the continuity and Navier–Stokes equations has to be adopted. However, the latter approach needs to resort to a numerical solution.

To illustrate the model predictions, liquid tin has been used as a model melt. The physical properties used in the calculations are specific density  $\rho = 6970 \text{ kg m}^{-3}$  and viscosity  $\mu = 0.00197 \text{ kg m}^{-1} \text{ s}^{-1}$  [7]. The operating parameters of the centrifugal atomization process are varied in the following ranges: volume flow rate,  $Q$ ,  $2.5 \times 10^{-6}$ – $2 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}$ ; cup rotation speed,  $\Omega$ , 50–1000 rps (revolutions per second); cup radius,  $r_o$ , 0.02–0.05 m; and cup slope angle,  $\alpha$ , 0–75°. In order to study the effects of the operating parameters individually, each parameter is usually varied at a time while the others are maintained constant as follows:  $Q = 1 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}$ ,  $\Omega = 250 \text{ rps}$ ,  $r_o = 0.03 \text{ m}$  and  $\alpha := 0^\circ$ . It should be noted that for





**Figure 3.** Variations of the degree of slippage and the velocity ratio with a compound parameter composed of liquid volume flow rate,  $Q$ , cup rotation speed,  $\Omega$ , cup radius,  $r_0$ , and cup slope angle,  $\alpha$ .

convenience the cup rotation speed shown in the illustrations is  $\Omega$ , with a unit of rps. In the calculations, however,  $\omega = 2\pi\Omega$ , with a unit of  $\text{radian s}^{-1}$ , is used instead.

The velocity of the melt at the point of leaving the atomizing cup, the ejection velocity, is an important indicator of the efficiency of atomization in centrifugal atomization. The ejection velocity also largely determines the cooling behaviour of the atomized liquid droplets because the velocity of the atomized droplets would be approximately equal to the ejection velocity. Although the ejection velocity is expected to be close to the tangential velocity of the rotating cup, a more accurate estimation needs to consider the liquid slippage on the cup as well as the slope-direction velocity of the melt at the cup edge. The magnitudes of slippage and slope-direction velocity can be conveniently evaluated by the degree of slippage,  $\phi$ , and the ratio of the slope-direction velocity to the absolute tangential velocity of the melt,  $\kappa = v_o/v_t$ . Equations (14) and (15) show that both  $\phi$  and  $\kappa$  are a function of a compound parameter,

$$c_1 = \left( \frac{Q^2 \cos \alpha}{\omega r_0^4} \right)^{1/3}. \quad (23)$$

They increase with increasing  $c_1$ , as shown in figure 3. In other words, increasing the volume flow rate or decreasing the cup rotation speed, cup radius or cup slope angle all lead to an increased degree of slippage,  $\phi$  and velocity ratio,  $\kappa$ . In the range of operating conditions examined in this paper, the degree of slippage is below 0.13 and the velocity ratio is below 0.26. In many cases, the peripheral velocity of the atomizing cup can be used as a first-order estimate of the ejection velocity.

The trajectory of the atomized droplets is an important factor for consideration in designing the atomizing chamber. It is expected not to be very different from the trajectory of the melt film beyond the atomizing cup. A small volume of liquid ejected from the atomizing cup would travel along a straight line, the direction of which is determined by the velocity ratio,  $\kappa$  and the cup slope angle,  $\alpha$ . However, the trajectory of an individual element of the melt is not convenient, and often not useful, for visualization of the flow development. The melt film needs to be considered as a whole. For a flat disc, the film would be a flat sheet if the gravitational effect is negligible. For a cup with a sloped wall, however, the vertical cross section of the film

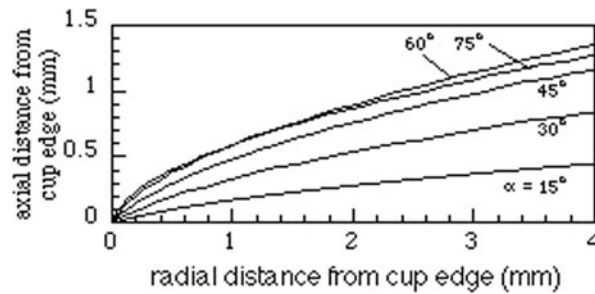


Figure 4. Variation of the shape of the liquid film off the atomizing cup with cup slope angle,  $\alpha$ .

would be a curve instead of a straight line. Figure 4 shows typical cross-sectional shapes of the melt film at different cup slope angles in the region immediately beyond the cup. For any melt film, the gradient of the curve at any radial distance is the ratio between the local axial and radial velocities:

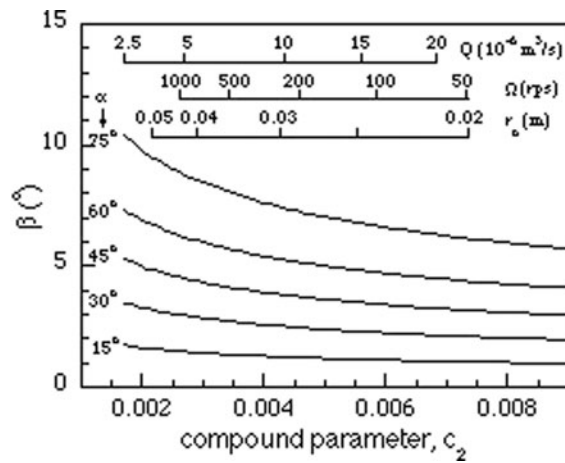
$$tg\beta = \frac{v_o \sin \alpha}{v_r}, \quad (24)$$

where  $\beta$  is the inclination angle of the curve, i.e. the angle between the tangent of the curve and the horizontal plane. Because the radial velocity,  $v_r$ , increases with the radial distance from the cup centre,  $r$ , the gradient decreases with the radial distance, as shown in equation (18). When the radial distance is several times greater than the cup radius,  $r_o$ , the radial velocity becomes nearly constant and the curve virtually becomes a straight line with a steady gradient and therefore a steady inclination angle. The steady inclination angle is a function of the processing parameters and decreases with a compound parameter,

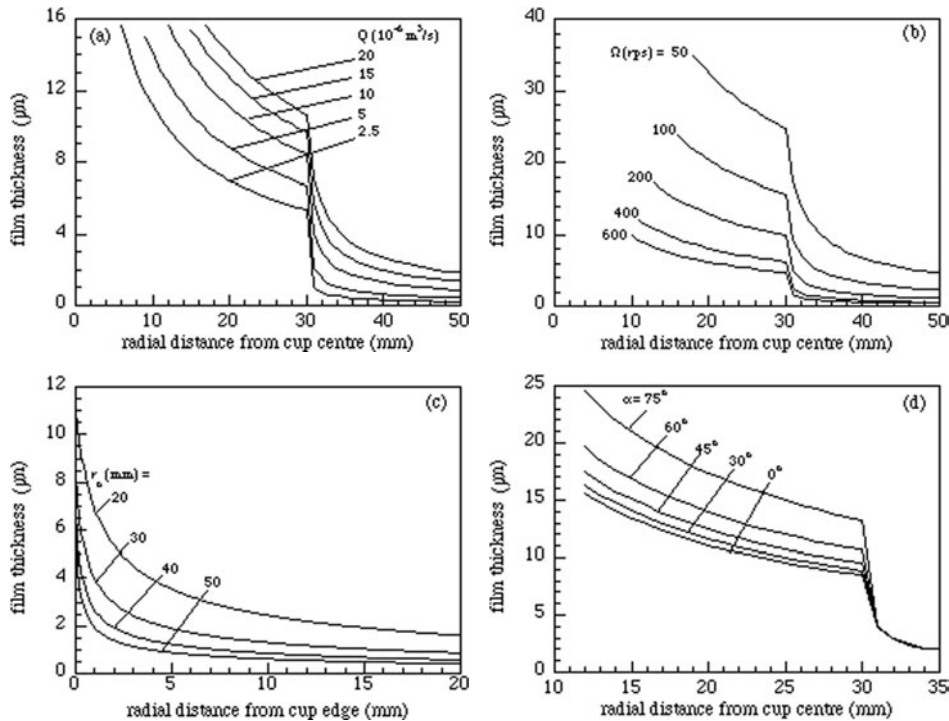
$$c_2 = \left( \frac{Q^2}{\omega r_o^4} \right)^{1/3}. \quad (25)$$

Figure 5 shows the variation in the steady inclination angle of the film,  $\beta$ , with  $c_2$ , as well as with the volume flow rate,  $Q$ , cup rotation speed,  $\Omega$ , and cup radius,  $r_o$ , under a series of cup slope angles  $\alpha$ . The steady inclination angle is usually small. In the range of operating conditions examined in this paper, it is normally much less than  $10^\circ$ .

The most important parameter in centrifugal atomization, whether for powder production or for spray deposition, is the size of the atomized droplets. Although the current model cannot predict the droplet size distribution directly, it can give an insight into the mechanism by which the operating condition affects the droplet size distribution. The development of the film thickness of a fully spreading melt on and off the atomizing cup under different operating conditions can be calculated using equations (9) and (20). Figures 6(a)–(d) show the variations in the film thickness with radial distance, either from the cup centre or from the cup edge, at different melt volume flow rates, cup rotation speeds, cup radii and cup slope angles. In each case, the parameters other than the one being varied are fixed at the typical values. It is shown that the film thickness decreases with increasing radial distance. There is a particularly rapid decrease in the film thickness when the melt has just been ejected from the cup edge. This implies that the disintegration of the film and thus the formation of individual droplets are likely to take place in a small region just off the cup edge. The film thickness of the melt at the edge of the atomizing cup is therefore a critical factor determining the sizes of the resultant droplets. Regardless of the disintegration mechanism, the diameters of the primary droplets are expected to be a few times the film thickness of the melt at the cup edge.



**Figure 5.** Variations of the steady inclination angle of the liquid film off the cup with a compound parameter composed of liquid volume flow rate,  $Q$ , cup rotation speed,  $\Omega$ , and cup radius,  $r_0$ , at different cup slope angles,  $\alpha$ .



**Figure 6.** Variations in film thickness with radial distance at different (a) liquid volume flow rates,  $Q$ , (b) cup rotation speeds,  $\Omega$ , (c) cup radii,  $r_0$ , and (d) cup slope angles,  $\alpha$ .

Figures 6(a)–(c) show that the film thickness of the melt at the cup edge decreases with decreasing volume flow rate, with increasing cup rotation speed and with increasing cup radius. The trends are generally consistent with the variations in the mean particle size when these parameters are changed as reported in the previous experimental studies [17].

Figure 6(d) shows that the film thickness of the melt at the cup edge increases with increasing cup slope angle, although not significantly. This is not consistent with the fact that a cup usually produces finer droplets than a flat disc under the same operating conditions [17]. Furthermore, the model cannot explain the fact that in certain cases there exist limits to the volume flow rate, cup rotation speed and cup radius, below or above which they no longer influence the droplet size distribution [17]. These inconsistencies indicate that the current model has some limitations. As the melts may not fully spread on the atomizing disc or cup in practice, the influence of spreading needs to be studied.

#### 4. Conclusion

A model has been developed for analysing the flow development of a fully spreading melt on and off the atomizing cup in centrifugal atomization. The model can be used to calculate the velocity and film thickness of the melt at any radial distance beyond the hydraulic jump as a function of melt volume flow rate, cup rotation speed, cup radius and cup slope angle, as well as to predict the shape of the spray off the cup. The model shows a sharp decrease in film thickness when the melt is ejected from the cup. This implies that the disintegration of the melt takes place in the region just off the cup edge and the film thickness at the cup edge is a critical factor determining the sizes of the resultant droplets. The film thickness at the cup edge is shown to decrease with decreasing volume flow rate, with increasing cup rotation speed, with increasing cup radius and with decreasing cup slope angle.

#### Acknowledgments

The author would like to thank Mr John Dunkley and Dr Jiwei Xie for some helpful discussions.

#### References

- [1] Fraser R P, Eisenklam E P and Dombrowski N 1957 *Br. Chem. Eng.* **2** 496
- [2] Masters K 1972 *Spraying Drying—An Introduction to Principles, Operational Practice and Applications* (London: Leonard Hill Books) p 93
- [3] Singer A R E and Kisakurek S E 1976 *Metals Technol.* **3** 565
- [4] Angers R 1992 *Powder Production and Spray Forming—Advances in Powder Metallurgy & Particulate Materials* vol 1, ed J M Capus and R M German (Princeton, NJ: Metal Powder Industries Federation) p 79
- [5] Osborne M G, Anderson I E, Funke K S and Verhoeven J D 1992 *Powder Production and Spray Forming—Advances in Powder Metallurgy & Particulate Materials* vol 1, ed J M Capus and R M German (Princeton, NJ: Metal Powder Industries Federation) p 89
- [6] Davies D R G and Singer A R E 1992 *Powder Production and Spray Forming—Advances in Powder Metallurgy & Particulate Materials* vol 1, ed J M Capus and R M German (Princeton, NJ: Metal Powder Industries Federation) p 301
- [7] Yule A J and Dunkley J J 1994 *Atomization of Melts for Powder Production and Spray Deposition* (Oxford: Clarendon Press)
- [8] Grant P S 1995 *Progr. Mater. Sci.* **39** 497
- [9] Jacobs M H, Young J M, Dowson A L and Ashworth M A 1995 *Advances in Powder Metallurgy and Particulate Materials* vol 2, ed M Philips and J Porter (Princeton, NJ: Metal Powder Industries Federation) p 7.57
- [10] Dowson A L, Duggan M A, Zhao Y Y, Jacobs M H and Young J M 1996 *Advances in Powder Metallurgy & Particulate Materials* vol 3, ed T M Cadle and K S Narasimhan (Princeton, NJ: Metal Powder Industries Federation) p 9.67
- [11] Dowson A L, Jacobs M H, Young J M, Zhao Y Y, Parsons B, Weale D and Bird M 1997 *Total Technology for Advanced Materials: Proc. of the 3rd Int. Conf. on Spray Forming* (Cardiff, UK: Osprey Metals Ltd) p 193
- [12] Li H P and Tsakirooulos P 1997 *J. Mat. Synthesis Process.* **5** 117

- 
- [13] Zhao Y Y, Jacobs M H and Dowson A L 1998 *Metal. Mat. Trans. B* **29** 1357
  - [14] Zhao Y Y, Dowson A L, Johnson T P, Young J M and Jacobs M H 1996 *Advances in Powder Metallurgy & Particulate Materials* vol 3, ed T M Cadle and K S Narasimhan (Princeton, NJ: Metal Powder Industries Federation) p 9.79
  - [15] Zhao Y Y, Dowson A L and Jacobs M H 2000 Modelling 2000 *Modelling Simul. Mater. Sci. Eng.* **8** 55
  - [16] White F M 1994 *Fluid Mechanics* 3rd edn (New York: McGraw-Hill) P 138
  - [17] Xie J W, Zhao Y Y and Dunkley J J 2004 Effect of processing parameters on powder size distribution and morphology in centrifugal atomization of tin *Powder Metall.* **47** 168–72