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Modelling of liquid flow after a hydraulic jump on a rotating disk prior to centrifugal atomization*

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Abstract. This paper describes a simplified numerical model which is used to calculate the height distribution, and the radial and tangential velocities of a liquid on a rotating disk after a hydraulic jump and prior to centrifugal atomization. The results obtained from this numerical model are compared with predictions made using previously derived 'hydraulic jump' and 'analytical' models. Calculations, in conjunction with experimental measurements relating to the trajectory of liquid flow on the atomizing disk, have shown that the numerical model can not only give a reasonable prediction of the hydraulic jump location, but also yields more accurate information regarding the variations in liquid height, and radial and tangential velocities. The model is ideally suited for engineering applications.

Nomenclature

$\mathrm{d}\vec{F}$	centrifugal force (vector)
$\mathrm{d}F_r$	radial component of centrifugal force
$\mathrm{d}F_\theta$	tangential component of centrifugal force
F	function related to radial velocity u
G	function related to tangential velocity v
H	function related to axial velocity w
h	liquid height on disk
I	maximum integer corresponding to h
i	integer variable corresponding to z
Q	liquid volume flow rate
Q_{sum}	intermediate variable of volume flow rate
$\vec{r}$	radial coordinate (vector)
r	radial coordinate in cylindrical system
u	radial liquid velocity relative to disk
$\bar{u}$	mean radial velocity
u_i	radial velocity of element i
$\vec{v}$	liquid velocity (vector)

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v tangential liquid velocity
$\bar{v}$ mean tangential velocity
v_i tangential velocity of element i
w axial liquid velocity
z axial coordinate in cylindrical system
Δz differential element in z
μ liquid viscosity
ν kinematic viscosity
θ tangential coordinate in cylindrical system
ρ specific density
τ viscous stress
ϕ degree of tangential slippage
$\vec{\omega}$ disk rotation speed (vector)
ω disk rotation speed (scalar)

1. Introduction

Centrifugal atomization and spray deposition are currently being developed for manufacturing powders and near-net-shape preforms from a range of advanced metallic materials [1–7]. The process utilizes a rapidly rotating disk to break up the liquid metal stream into a spray of droplets which either solidify in flight to form high-quality powders or deposit onto a substrate to form microstructurally refined and chemically homogeneous preforms. The powder characteristics, deposit microstructures and associated mechanical properties are determined primarily by the size, temperature and velocity distribution of the atomized droplets and by the rate at which they impinge on the substrate. The atomization process in turn is dependent on the behaviour of the liquid as it impacts with, and flows across, the atomizing disk.

In centrifugal atomization of liquid metals, the atomizing disk usually rotates at a very high speed, typically in excess of 5000 rpm, in order to yield droplet size distributions with maxima less than 200 μm. After the liquid metal stream impinges on the disk, the high centrifugal force drives the liquid to flow radially at accelerating tangential and radial velocities. Such increases in radial velocity are not, however, infinitely sustainable and the flow is generally accompanied by a hydraulic jump, an annular discontinuity in the flow pattern which is manifested by an abrupt increase in the height of liquid metal on the disk, and by a corresponding reduction in the radial velocity [8]. The hydraulic jump radius is usually smaller than the atomizing disk radius in the processing of liquid metals. This paper is concerned solely with the liquid flow after the hydraulic jump because it is this process that has a major influence on the subsequent atomization, and it is our ultimate goal to model and understand the atomization process.

The precise description of liquid flow on a rapidly rotating disk requires the simultaneous solution of the governing equations of mass, momentum and energy conservation. As a result exact solutions can only be achieved through the use of complex numerical procedures. However, since the problem involves a free surface, strong swirling motions and, perhaps most importantly, a hydraulic jump, it is difficult to solve using current commercially available computational fluid dynamics (CFD) packages. In the absence of such packages, approximate analytical and empirical solutions have been sought, both to give a semi-quantitative mechanistic insight into the effects of different atomizing conditions, and to provide an engineering tool for process development.

Two previous models detailing the flow of liquid metal on a disk during centrifugal atomization have been developed by the authors: a 'hydraulic jump model' [8] in which velocity trial functions are used in order to satisfy the prime boundary conditions for the conservation of mass and momentum, and an 'analytical model' [9] in which the effects of tangential slippage are largely ignored and the viscous and centrifugal forces are balanced. Whilst the analytical model gives a reasonable approximation of the liquid metal profile on the disk at large disk radii where the thickness of liquid metal is small and where tangential slippage is negligible, large errors can arise at smaller disk radii where slippage becomes significant. Similarly the hydraulic jump model approach yields considerable errors when the thickness of liquid metal on the disk is very small, as is commonly observed at the edge of the disk. In order to address these limitations the current paper describes a simplified numerical model which is capable of predicting the height, and the radial and tangential velocity distributions of a liquid metal on a rotating disk immediately after a hydraulic jump and prior to disintegration at the edge of the atomizing disk. The model is validated by both experimental measurements and comparison with predictions made by the previously developed hydraulic jump and analytical models.

2. Model

The height profile of a liquid metal on a rotating disk, and the centrifugal and viscous forces exerted on a small volume of the liquid prior to centrifugal atomization, are shown schematically in figure 1. The liquid metal flows from a nozzle onto the centre of the disk, and then spreads rapidly towards the edge due to the action of gravitational and centrifugal forces. Assuming that a hydraulic jump takes place at a radius r_c, the liquid flow after the jump is mainly controlled by the rotating disk. The centrifugal force accelerates the liquid metal both tangentially and radially to a high velocity at the periphery of the disk and atomizes the liquid into a spray of droplets.

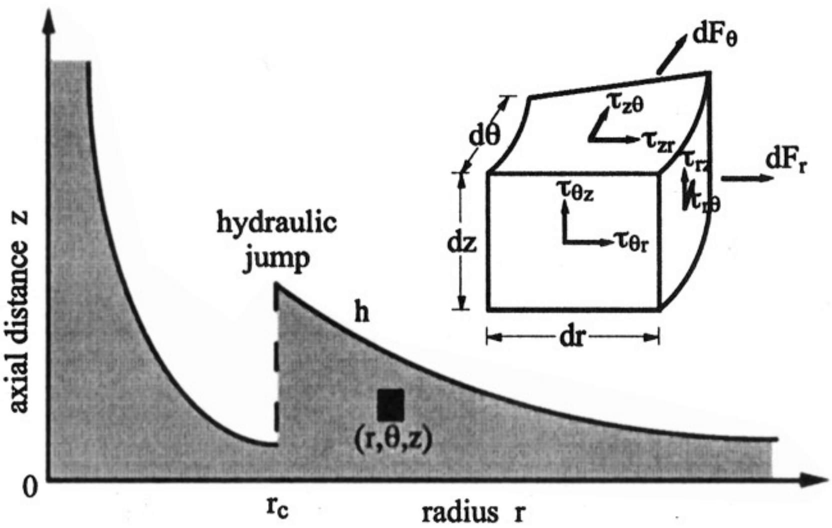

Figure 1. Schematic diagram of the height profile of the liquid metal on an atomizing disk, assuming a hydraulic jump takes place at r_c, and the centrifugal and viscous forces exerted on a small volume of the liquid.

Considering an infinitesimally small volume of liquid metal at a point (r, θ, z) in the liquid flow after a hydraulic jump as shown in figure 1, the centrifugal force $\mathrm{d}\vec{F}$ is given by [10]

$$\mathrm{d}\vec{F} = -\rho[2\vec{\omega} \times \vec{v} + \vec{\omega} \times (\vec{\omega} \times \vec{r})]r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z \tag{1}$$

where ρ is the specific density of the liquid metal, $\vec{\omega}$ is the rotation speed of the disk (vector), $\vec{v}$ is the velocity of the liquid metal relative to the rotating disk (vector), $\vec{r}$ is the radial coordinate (vector), and r, θ and z are the radial, tangential and axial coordinates, respectively, in the cylindrical coordinate system. The radial and tangential components of the centrifugal force are therefore

$$\begin{aligned} \mathrm{d}F_r &= \rho(\omega^2 r + 2\omega v)r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z \\ \mathrm{d}F_\theta &= -2\rho\omega u r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z \end{aligned} \tag{2}$$

where $\mathrm{d}F_r$ and $\mathrm{d}F_\theta$ are radial and tangential components of the centrifugal force, respectively, ω is the rotation speed of the atomizing disk (scalar), and u and v are the radial and tangential components, respectively, of the liquid velocity relative to the disk.

Assuming that the liquid flow is axisymmetrical and Newtonian with a constant viscosity, the viscous stresses at the surfaces are expressed in relation to the velocities [11]:

$$\begin{aligned} \tau_{r\theta} &= \tau_{\theta r} = \mu r \frac{\partial}{\partial r}\left(\frac{v}{r}\right) \\ \tau_{\theta z} &= \tau_{z\theta} = \mu \frac{\partial v}{\partial z} \\ \tau_{zr} &= \tau_{rz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \end{aligned} \tag{3}$$

where τ is the stress, the first letter of the subscript denotes the direction perpendicular to the surface, the second letter of the subscript is the direction of the stress, μ is the viscosity of the liquid and w is the axial component of the velocity.

For steady-state flow, the centrifugal forces exerted on the volume are balanced by the viscous forces due to the difference of the viscous stress at the opposite surfaces. In the radial and tangential directions therefore

$$\begin{aligned} \mathrm{d}F_r + \mathrm{d}\tau_{zr} r\,\mathrm{d}r\,\mathrm{d}\theta + \mathrm{d}\tau_{\theta r}\,\mathrm{d}r\,\mathrm{d}z &= 0 \\ \mathrm{d}F_\theta + \mathrm{d}\tau_{z\theta} r\,\mathrm{d}r\,\mathrm{d}\theta + \mathrm{d}\tau_{r\theta} r\,\mathrm{d}\theta\,\mathrm{d}z &= 0. \end{aligned} \tag{4}$$

Kámán has shown that the velocity components of liquid flow above a rotating disk relative to the disk can be approximated by

$$u = \omega r F\left(\sqrt{\frac{\omega}{\nu}}z\right) \qquad v = \omega r\left[G\left(\sqrt{\frac{\omega}{\nu}}z\right) - 1\right] \qquad w = \sqrt{\nu\omega}H\left(\sqrt{\frac{\omega}{\nu}}z\right)$$

where F, G and H are functions which satisfy the momentum and mass conservation equations of the flow [12]. In other words, the tangential velocity v is proportional to the radius r, whereas the axial velocity w is independent of the radius r. As can be seen from equation (3), $\tau_{r\theta}$ and $\tau_{\theta r}$ can be neglected, and $\tau_{zr}$and τ_{rz} can be reduced to be functions of radial velocity u only. The problem is then simplified into a one-dimensional problem. Combining equations (2)–(4) and the mass conservation equation gives the governing equations:

$$\begin{aligned} &\nu\frac{\partial^2 u}{\partial z^2} + \omega^2 r + 2\omega v = 0 \\ &\nu\frac{\partial^2 v}{\partial z^2} - 2\omega u = 0 \\ &\int_0^h 2\pi r u\,\mathrm{d}z = Q \end{aligned} \tag{5}$$

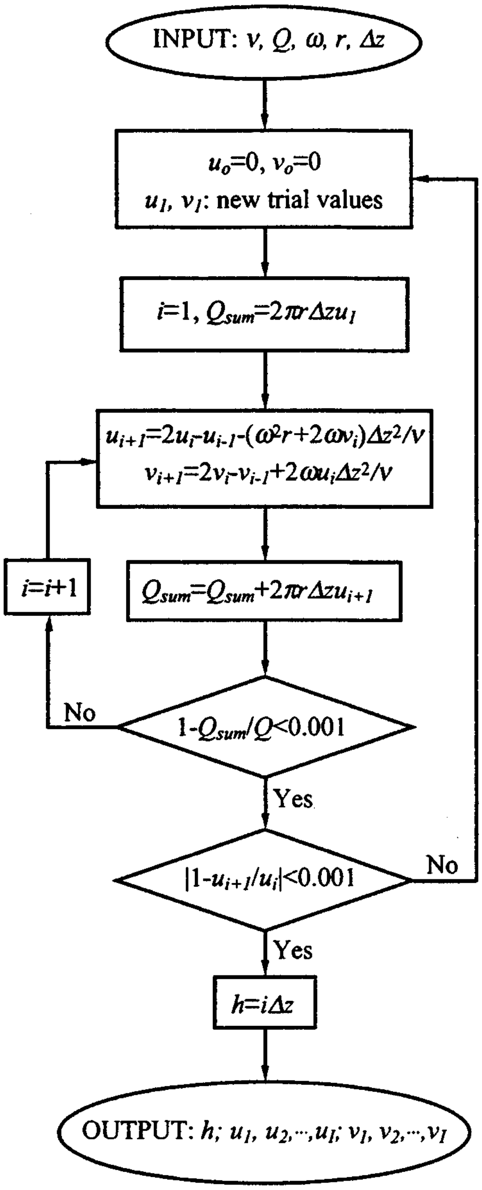

Figure 2. Block diagram of the computer program for calculations of the liquid height and velocities.

where $\nu = \mu/\rho$ is the kinematic viscosity of the liquid, h is the liquid height at radius r and Q is the volume flow rate of the liquid at any radius r, which is a constant for steady-state flow and equals the rate of liquid flow from the nozzle to the disk. Because there is no slippage at the interface between the liquid and the disk, the liquid velocity relative to the disk is zero. Assuming that the effect of surface tension of the liquid is negligible, the viscous stresses at the top surface of the liquid are approximately zero. Therefore, the boundary conditions are: $z = 0$, $u = 0$, $v = 0$; $z = h$, $\partial u/\partial z = 0$, $\partial v/\partial z = 0$.

For computer or numerical modelling, the above equations can be discretized as

$$\frac{\nu}{\Delta z^2}(u_{i+1} + u_{i-1} - 2u_i) + \omega^2 r + 2\omega v_i = 0$$
$$\frac{\nu}{\Delta z^2}(v_{i+1} + v_{i-1} - 2v_i) - 2\omega u_i = 0$$
$$2\pi r \Delta z \sum_{i=1}^{I} u_i = Q \tag{6}$$

where Δz is the differential element and i is an integer between 1 and I. The boundary conditions are: $u_0 = 0$, $v_0 = 0$; $u_{I-1} = u_I$, $v_{I-1} = v_I$. Figure 2 shows a block diagram of a FORTRAN program used to solve the above equations, in which Q_{sum} is an intermediate variable of volume flow rate. Given the liquid kinematic viscosity ν, volume flow rate Q, disk rotation speed ω and a radial position r (where $r > r_c$), the computer program can give the liquid height h, the radial velocity u and the tangential velocity v as functions of the axial distance z from the disk, converging to an accuracy of within 0.1%.

3. Calculations and discussions

Calculated variations in liquid height with radius after the hydraulic jump under different liquid volume flow rates, disk rotation speeds and liquid kinematic viscosities are shown in figures 3(*a*)–(*c*), respectively, and are compared with similar predictions obtained using an analytical model developed in a previous paper [9]. For the purpose of experimental validation, a liquid specific density $\rho = 3800$ kg m^{-3} and a viscosity $\mu = 0.0048$ kg m^{-1} s^{-1} (kinematic viscosity $\nu = 1.263 \times 10^{-6}$ m^2 s^{-1}) have been assumed which are consistent with liquid Ti–48Al–2Mn–2Nb at a superheat of 50 °C above the melting point [9]. With decreasing liquid temperature during atomization, however, the kinematic viscosity increases. For this reason, and in order to determine the sensitivity of the analysis, for each parameter varied in figures 3(*a*)–(*c*) the remaining two parameters are maintained constant: volume flow rate $Q = 5 \times 10^{-5}$ m^3 s^{-1}, disk rotation speed $\omega = 1000$ radian s^{-1} or kinematic viscosity $\nu = 1.263 \times 10^{-6}$ m^2 s^{-1}. The analytical calculations were carried out only for a radius r greater than the critical hydraulic jump radius r_{c}, which is given by the hydraulic jump model proposed by Zhao *et al* [8]. The hydraulic jump location predicted by the numerical model is the radius at which the radial velocity tends to zero or the liquid height tends to infinity.

Figure 3 shows that the hydraulic jump radius increases with increasing liquid volume flow rate, decreasing disk rotation speed and decreasing liquid kinematic viscosity, which is consistent with predictions made using the hydraulic jump model. The jump radius calculated by the present numerical model is generally greater than that calculated by the jump model. For a volume flow rate of between 2×10^{-5} and 1×10^{-4} m^3 s^{-1}, a disk rotation speed of between 400 and 2000 radian s^{-1} and a kinematic viscosity of between 4×10^{-7} and 2×10^{-6} m^2 s^{-1}, the hydraulic jump radius predicted by the jump model and the numerical model ranges from 0.013 to 0.029 m and from 0.018 to 0.040 m, respectively.

Both analytical and numerical calculations in figure 3 show that, for a fixed processing condition, the liquid height decreases gradually on moving radially outwards from the jump radius. For any given radial position after the jump, the height of liquid metal on the disk increases with increasing volume flow rate, with decreasing disk rotation speed and with increasing kinematic viscosity. At a radius much greater than the hydraulic jump radius, the differences between the analytical and numerical predictions are usually small. At a radius $r = 0.1$ m, the liquid height increases from 10.66 to 18.48 μm, from 9.14 to 26.91 μm, and

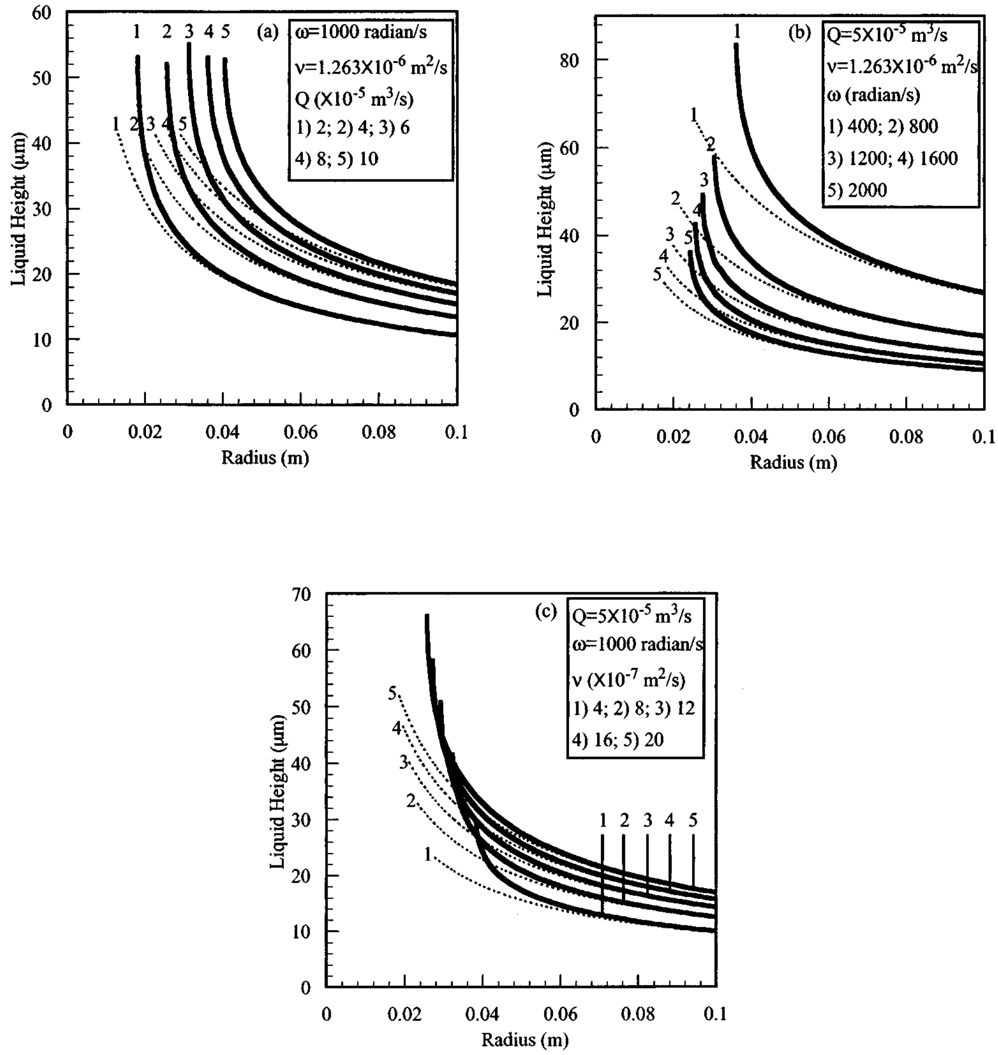

Figure 3. Variation of liquid height after hydraulic jump with different (*a*) volume flow rates, (*b*) disk rotation speeds and (*c*) kinematic viscosities, calculated by the analytical model [9] (——) and the numerical model (······).

from 9.98 to 16.91 μm with increasing volume flow rate from 2×10^{-5} to 1×10^{-4} m^3 s^{-1}, with decreasing disk rotation speed from 2000 to 400 radian s^{-1} and with increasing kinematic viscosity from 4×10^{-7} to 2×10^{-6} m^2 s^{-1}, respectively.

The radial and tangential components of the liquid velocity on the atomizing disk are functions of the radial and axial distances from the disk. At the interface between the liquid metal and the disk, the liquid has a zero velocity relative to the atomizing disk. However, as the axial distance is increased, both the radial and tangential velocities relative to the disk increase. The important parameters in terms of centrifugal atomization would appear to be the mean radial and tangential velocities as a function of disk radius, which can be expressed as $\bar{u} = \int_0^h u\, \mathrm{d}z/h$ and $\bar{v} = \int_0^h v\, \mathrm{d}z/h$, respectively. Figures 4(*a*)–(*c*) show the variations in mean radial velocity with radius after the hydraulic jump under different volume flow

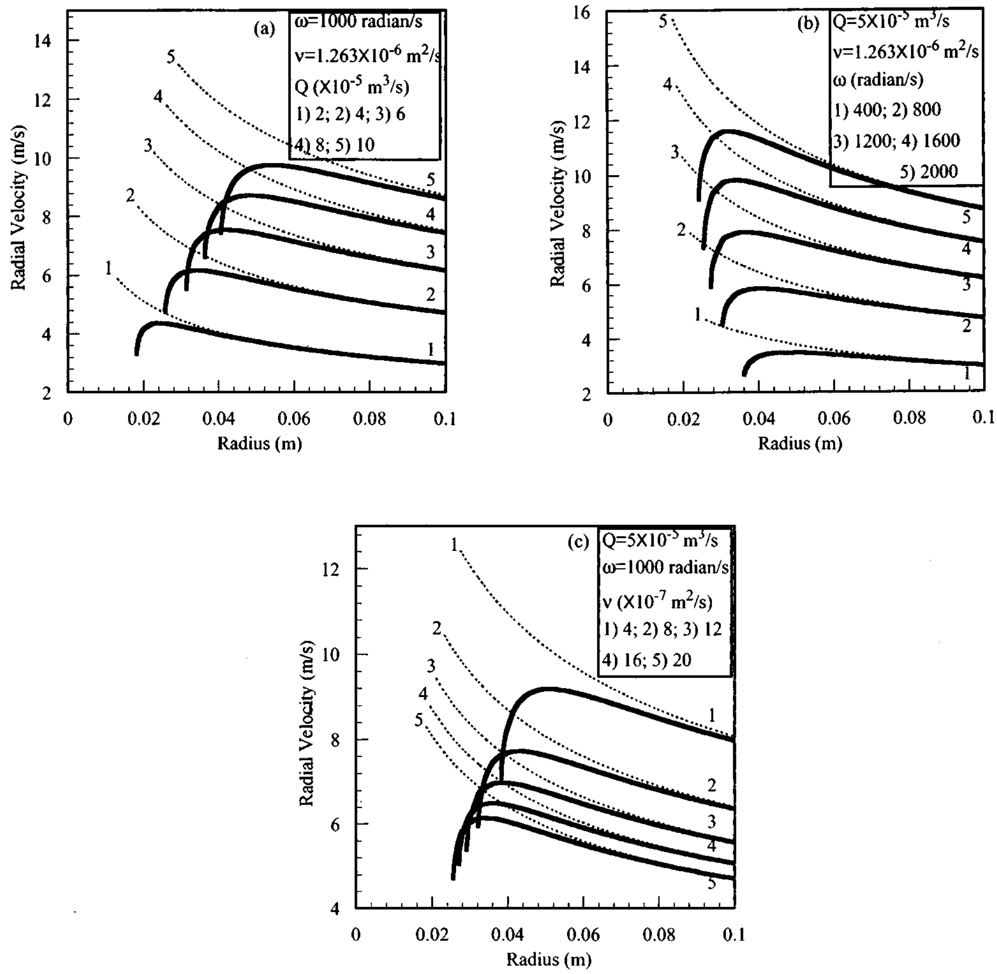

Figure 4. Variation of mean radial velocity after hydraulic jump with different (*a*) volume flow rates, (*b*) disk rotation speeds and (*c*) kinematic viscosities, calculated by the analytical model [9] (——) and the numerical model (· · · · · ·).

rates, disk rotation speeds and kinematic viscosities, respectively, with the remaining two parameters maintained constant: volume flow rate $Q = 5 \times 10^{-5}$ m^3 s^{-1}, disk rotation speed $\omega = 1000$ radian s^{-1} or kinematic viscosity $\nu = 1.263 \times 10^{-6}$ m^2 s^{-1}. Again analytical predictions have been included for comparison. The numerical calculations show that, for a fixed processing condition, the mean radial velocity first increases rapidly and then decreases steadily with increasing radius after the jump. The analytical and numerical predictions deviate considerably close to the hydraulic jump, but gradually converge with increasing radius. For any given radial position, the mean radial velocity increases with increasing volume flow rate, with increasing disk rotation speed and with decreasing kinematic viscosity. At a radius of 0.1 m, the mean radial velocity of the liquid increases from 2.99 to 8.61 m s^{-1}, from 2.96 to 8.71 m s^{-1} and from 4.71 to 7.98 m s^{-1} with increasing volume flow rate from 2×10^{-5} to 1×10^{-4} m^3 s^{-1}, with increasing disk rotation speed from 400 to 2000 radian s^{-1} and with decreasing kinematic viscosity from 2×10^{-6} to 4×10^{-7} m^2 s^{-1}, respectively.

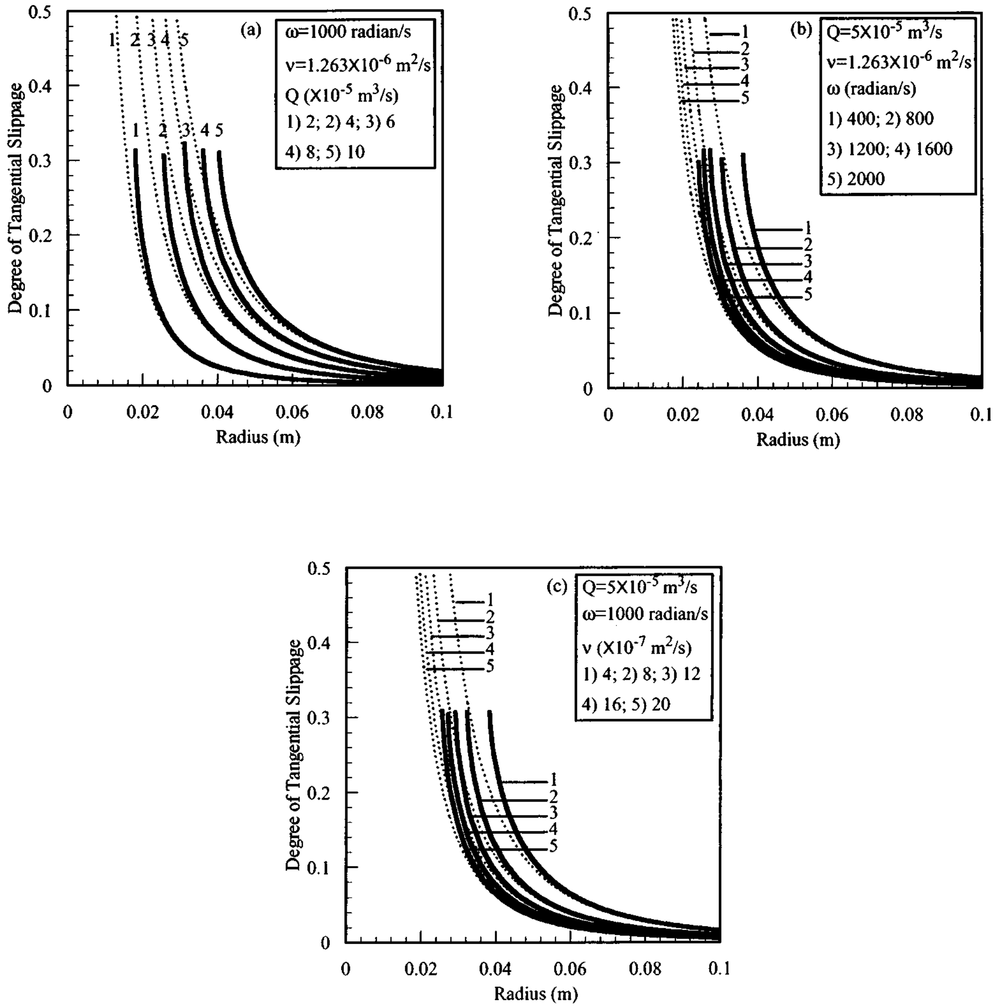

Figure 5. Variation of degree of tangential slippage after hydraulic jump with different (*a*) volume flow rates, (*b*) disk rotation speeds and (*c*) kinematic viscosities, calculated by the analytical model [9] (——) and the numerical model (······).

The mean tangential velocity of the liquid is smaller than, but usually close to, that of the disk. On this basis, it is more convenient to describe the deviation of the tangential velocity of the liquid from that of the disk by a parameter designated as the degree of tangential slippage $\phi = 1 - (\bar{v}/\omega r)$, which is the ratio between the mean tangential velocity of the liquid relative to the disk and the absolute velocity of the disk [8]. Figures 5(*a*)–(*c*) show the variations in the degree of slippage with radius for different volume flow rates, disk rotation speeds and kinematic viscosities, respectively, with the remaining two parameters maintained constant: volume flow rate $Q = 5 \times 10^{-5}$ m^3 s^{-1}, disk rotation speed $\omega = 1000$ radian s^{-1} or kinematic viscosity $\nu = 1.263 \times 10^{-6}$ m^2 s^{-1}. Immediately following the hydraulic jump the degree of slippage is very high and as a consequence the mean tangential velocity is low. However, with the liquid flowing radially outwards from the jump position, the degree of slippage decreases, first rapidly and then relatively slowly. At any specified radius, the degree of slippage increases

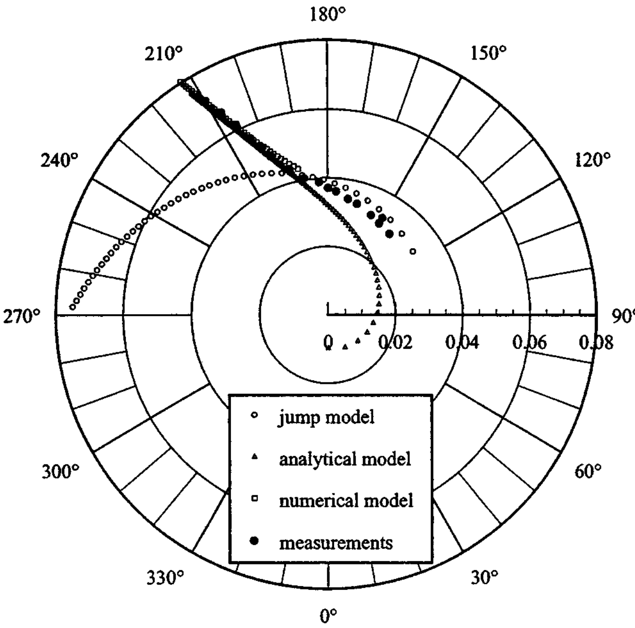

Figure 6. Measured and calculated flow lines of the liquid metal on the atomizing disk.

with increasing volume flow rate, with decreasing disk rotation speed and with decreasing kinematic viscosity. For the range of processing conditions commonly encountered in the centrifugal atomization of liquid metals, the degree of slippage ranges from 0.014 to 0.132 at a radius $r = 0.05$ m, but falls below 0.019 at $r = 0.1$ m. Because the analytical model [9] ignores the effects of tangential slippage, it gives considerable error close to the hydraulic jump region where slippage becomes significant. When tangential slippage is small (typically at large disk radii), the analytical model gives a good approximation of the liquid metal profile and velocities on the disk.

Recent studies conducted by Zhao *et al* [8, 9] have shown that the flow lines observed on the skull which forms on the atomizing disk during atomization can be related directly to the flow of liquid metal on the disk, and can be used to evaluate the accuracy of the velocity calculations. Experimental measurements of the flow lines developed during the atomization of liquid Ti–48Al–2Mn–2Nb have been reported in previous papers [8, 9]. The atomizing conditions included a volume flow rate $Q = 6.096 \times 10^{-5}$ m^3 s^{-1} and a disk rotation speed $\omega = 314.16$ radian s^{-1} assuming that the kinematic viscosity $\nu = 1.263 \times 10^{-6}$ m^2 s^{-1}. Figure 6 compares the experimental measurements with the predicted values calculated using the present numerical model, the jump model [8] and the analytical model [9] for the trajectory of the liquid metal after the hydraulic jump. The jump model gives good predictions of the hydraulic jump location r_c and the trajectory close to the jump or at radii less than 0.04 m. Large deviations are, however, observed as the radius is increased. In contrast, the analytical model yields a better prediction at radii greater than 0.04 m but shows considerable deviation when the radius is smaller than this value due to the high degree of slippage immediately after the jump. The analytical model, however, cannot predict the occurrence of the hydraulic jump and therefore gives meaningless values before the hydraulic jump. The numerical model not only gives more accurate predictions than the analytical model at large radii but can also predict the location of the hydraulic jump, although this is at a slightly greater radius than that predicted by the hydraulic jump model. Overall, the simplified numerical model is more suitable for engineering applications.

4. Conclusion

A simplified numerical model has been developed to predict the height distribution, and the radial and tangential velocities of a liquid metal on a rotating disk following a hydraulic jump as functions of the atomization conditions. The model assumes a balance between viscous and centrifugal forces and takes into consideration the coupling between the radial and tangential velocities. Compared with previously developed 'hydraulic jump' and 'analytical' models, the numerical model not only predicts the location of the hydraulic jump but also gives a more accurate approximation of the liquid height, and the radial and tangential velocities after the jump. The model has been validated using experimental measurements of the liquid flow trajectory on an atomizing disk during the centrifugal atomization of liquid Ti–48Al–2Mn–2Nb.

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