

Laminar viscoelastic flow over a backward-facing step

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ABSTRACT

The results are reported of an experimental investigation of the laminar flow of a series of viscoelastic liquids, 0.05%, 0.1% and 0.4% w/w/ aqueous solutions of polyacrylamide (Seperan AP 273 E), through a plane sudden expansion, of expansion ratio, $R=D/d=1.43$, immediately preceded by a smooth contraction. The two lower concentrations, 0.05% and 0.1%, are at essentially the same Reynolds number (ca. 120, based on a characteristic viscosity corresponding to U_B/h) and demonstrate the effect of increasing concentration on the flow field for a situation with a relatively high degree of inertia (at least in comparison to previous studies which were for very low (~ 1) Re). As concentration increases the reattachment length is decreased from 7.25 h for 0.05% PAA to less than 3h for 0.1% PAA, as are the magnitudes of the recirculating velocities i.e. recirculation is suppressed. For the highest concentration (0.4% PAA, $Re \approx 5$) the flow field downstream of the expansion is devoid of recirculation and this is a consequence of the viscoelasticity of the liquid damping out vortex activity. The effect of the smooth contraction that precedes the expansion is more thought provoking. The velocity profile produced by the contraction is intriguing, with velocity overshoots near the sidewall for 0.05% and 0.1% PAA and being more pyramidal in shape for 0.4% PAA as opposed to the uniform shape of a Newtonian fluid-flow, and attempts are made to understand the physical processes causing these profiles and to relate this to degree of viscoelasticity of the liquid.

INTRODUCTION

For laminar Newtonian fluid flow through a plane sudden expansion (PSE) it is well known that when the expansion ratio $R=D/d$ exceeds 1.5 the flow field downstream of the expansion becomes asymmetric above a critical Reynolds number. This has been observed both experimentally (e.g. [1]) and numerically (e.g. [2]). The critical Reynolds number (Re_{CR}) at which this switch to asymmetric flow is observed is dependent upon both the upstream flow conditions (e.g. fully-developed or uniform velocity profile) and the aspect ratio of the expansion, $A_d=w/d$ or $A_h=w/h$.

In contrast to the situation for Newtonian fluids, the laminar flow of non-Newtonian, viscoelastic, liquids through sudden expansions has received scant attention in the literature and is mainly restricted to a handful of papers at very low Reynolds numbers, where the flow remains symmetric, and has involved flow visualisation [3], [4]) and theoretical modelling ([4], [5] and [6]) but no detailed measurements of the flowfield. The general consensus from all of these works is that as the viscoelastic fluid flows through a sudden expansion it releases some of its stored energy, resulting in an expansion of the main flow and compression of the recirculation

region in a similar manner to the well-known phenomenon of die-swell.

The prediction of viscoelastic flow in complex geometries such as sudden expansions has both scientific interest and industrial relevance. The verification of the theoretical models to predict these flows has always been at best qualitative in nature and it is one of the purposes of the present work to address this deficit by providing detailed rheological and velocity data to enable quantitative validation of these models.

EXPERIMENTAL RIG

The flow loop used for the present experiments was a modified version of that used by [7] for their square-duct investigation. The square-duct consisted of ten stainless steel modules each of length 1.2m and with an internal cross section of side length 80 mm. The plane sudden expansion, for which the key dimensions are given in **Figure 1**, replaced one of the existing modules 9.6 m from the inlet connection. This arrangement provides a length of 120 hydraulic diameters for the flow to become fully developed. The duct width w throughout is 80 mm, the inlet height d is 28 mm and the step height h is 6 mm. The downstream duct height D is 40 mm. The

expansion was preceded by a short (60 mm in length), smooth contraction (50 mm radius followed by 11 mm radius). This produces an expansion ratio ($R=D/d$) of 1.43 and an aspect ratio (w/h) of 13.33 and the expansion acts essentially as a double backward-facing step i.e. the flow remains symmetric. The sidewalls of the expansion were made of borosilicate glass to permit velocity measurements using a laser Doppler anemometer. Spanwise distributions of mean axial (U) were obtained upstream of the expansion at $x/h=-8.33$ and at inlet $x/h=0$. Transverse distributions of U were also obtained from traverses at 10 axial locations (corresponding to x/h values of 0,1,2,3,4,5,6,7,8 and 10).

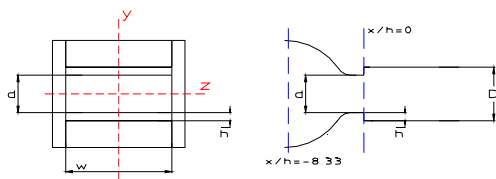


Figure 1: Schematic of backward-facing step.

RHEOLOGY

The working fluids used in this investigation were aqueous solutions (0.05, 0.1 and 0.4%) of w/w polyacrylamide (PAA): Seperan AP273 E supplied by SNF UK limited. The solvent used was filtered tap water with 100 ppm of 40% formaldehyde solution added to prevent bacterial degradation. Approximately 0.25 gm of Timiron seeding particles were added to the fluid (total volume of fluid being 575 litres) to improve the LDA signal quality. PAA was chosen as it is highly viscoelastic, is optically transparent (thereby permitting LDA measurements) and has been used extensively in previous investigations in the same laboratory (see e.g. [7]).

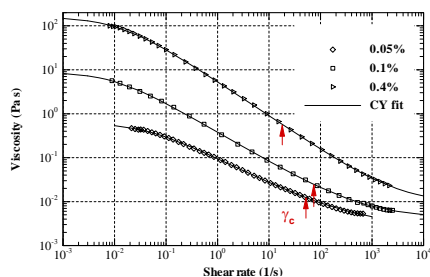


Figure 2: Viscosity versus shear rate

The measured viscosity versus shear rate data for the three concentrations of PAA is shown in **Figure 2** together with the Carreau-Yasuda model fits.

RESULTS AND DISCUSSION

Flow through smooth contraction

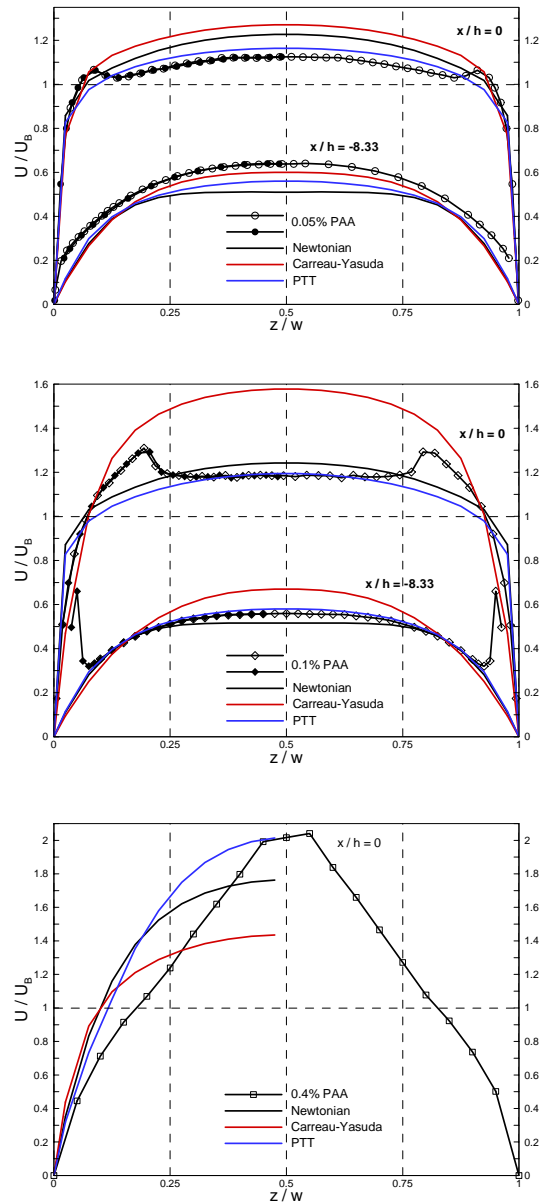


Figure 3: Spanwise variation of axial velocity (U/U_B) profiles within smooth contraction.

The spanwise variation of the axial velocity profiles, i.e. in the XZ-centreplane, both within the smooth contraction (at $x/h=-8.33$) and at inlet ($x/h=0$) are shown above in **Figure 3**. The two lower concentrations, 0.05% and 0.1%, are at essentially the same Reynolds number (ca. 120, based on the viscosity corresponding to a characteristic shear rate of U_B/h and highlighted in **Figure 2**) whilst the 0.4% PAA is at a Reynolds

number of about 5. For 0.05% and 0.1% PAA it can be observed that close to the sidewalls a velocity overshoot develops and moves towards the XY-centreplane (i.e. $z/w = 0.5$) as the flow progresses through the contraction (c.f. 0.4% PAA where the overshoots seem to have completely merged). Three different numerical simulations (Newtonian, inelastic GNF and viscoelastic PTT) fail to capture, even in a qualitative sense, this behaviour.

Flow downstream of expansion

Axial velocity profiles downstream of the backward-facing step in the XY-centreplane are shown in Figure 4.

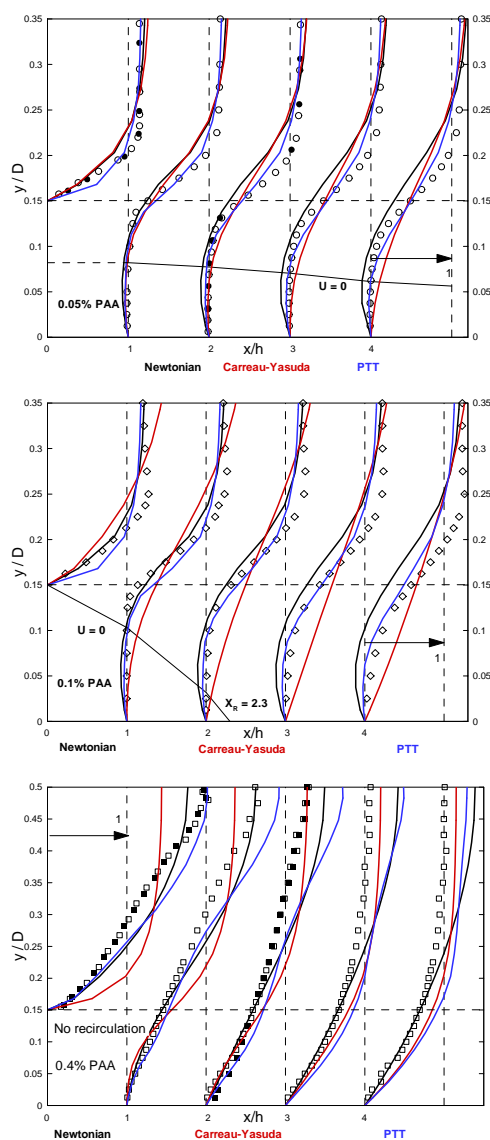


Figure 4: Axial velocity profiles downstream of backward-facing step.

Downstream of the expansion the effect of increasing viscoelasticity is for a general suppression of recirculation: at identical Reynolds number the recirculation region reduces from 7.25 h for 0.05% PAA to 2.3 h for 0.1% PAA. The combination of lower Reynolds number (≈ 5) and further increased viscoelasticity suppress recirculation entirely for the 0.4% PAA flow.

Agreement with the viscoelastic (PTT) simulations is better for the 0.05% PAA fluid flow than with the flow through the smooth contraction although discrepancies are still significant. This lack of quantitative agreement, both for this and the higher concentrations, may well be a consequence of the poor agreement with the inlet velocity profiles (i.e. the inability of the simulations to predict the flow through the smooth contraction).

CONCLUSIONS

The flow through the smooth contraction becomes increasingly three-dimensional (but symmetrical about the XY-centreplane) and complex with increasing PAA concentration. The simulations fail to predict the velocity overshoot near the sidewalls.

The flow over the backward-facing step always remains symmetrical with respect to the XZ-centreplane. The 0.05% PAA flow is predicted reasonably well by the PTT model presumably a consequence of it being more two-dimensional. At higher concentrations the profiles are not predicted well by any model although this may be a consequence of the poor agreement with the simulation through the contraction and hence with the resulting inlet velocity profiles.

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