Laminar viscoelastic flow through a plane sudden expansion

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Abstract

The results are reported of an experimental investigation of the laminar flow of a viscoelastic liquid, a 0.4% aqueous solution of polyacrylamide (Seperan AP 273 E), through a plane sudden expansion, of expansion ratio R=D/d=1.43, immediately preceded by a smooth contraction. The flow field downstream of the expansion is devoid of recirculation and this is a consequence of the viscoelasticity of the fluid damping out vortex activity. The effect of the smooth contraction is more thought provoking. The velocity profile produced by the contraction is intriguing and attempts are made to understand the physical processes generating this profile and to relate this to the viscoelasticity of the liquid.

1. Introduction

The laminar flow of non-Newtonian, viscoelastic, liquids through sudden expansions has received scant attention in the literature and is restricted to a handful of papers concerned with flow visualisation and theoretical modelling, primarily at very low Reynolds numbers.

The first work to investigate expansive flows of viscoelastic, non-Newtonian liquids is that of Halmos and Boger (1976). Streak photography was used for flow visualisation while a flash technique was used for point centreline velocity measurements. In earlier work, Halmos et al (1975a and b), these authors presented a numerical solution with experimental verification for the flow of inelastic, power-law fluids through an axisymmetric sudden expansion. Halmos and Boger (1976) concluded that as a viscoelastic fluid flows through a sudden expansion it releases some of its stored energy, resulting in an expansion of the main flow and compression of the secondary cell (recirculation region).

Townsend and Walters (1994) used flow visualisation to observe the flow field downstream of both a two-dimensional and a three-dimensional expansion for a 0.15% aqueous solution of polyacrylamide. The conclusion drawn from the study was that the viscoelasticity of the polymer solution damped out the vortex activity (which is generated by fluid inertia) causing any recirculating fluid to be pushed into the corners of the expansion. A theoretical model was also developed to simulate the flow field numerically. The results were in good physical agreement with the observed flow visualisation but only qualitative in nature due to the lack of quantitative rheological and velocity data.

The numerical studies of Darwish et al (1992) and later Missirlis et al (1998) are very similar. Both used a finite-volume technique to simulate the flow of a viscoelastic liquid through a two-dimensional 4:1 plane sudden expansion. Neither validated their work by comparison with experimental data but inferred verification by using grid refinement to obtain grid-independent results. Missirlis et al showed that the suppression of vortex activity is related to the Deborah number ($De = t/T$ where T is a characteristic time of the deformation process being observed and t is a characteristic time of the material). They showed that as De is increased beyond a critical value of 3.0 the recirculation zone is completely eliminated.

The experiments of Townsend and Walters (1994) were also used as the basis for comparison in the numerical simulation work of Baloch et al (1996). Expansion flows were modelled in two and three dimensions using a class of constitutive models due to Phan-Thien and Tanner (1977). Once again good qualitative agreement is seen with the experimental visualisations and the conclusion again drawn that viscoelasticity suppresses vortex activity and that this suppression is linked to the phenomenon of die swell.

The prediction of viscoelastic flow in complex geometries such as sudden expansions has both scientific interest and industrial relevance. The verification of the theoretical models to predict these flows has always been at best qualitative in nature and it is one of the purposes of the present work to address this deficit by providing detailed rheological and velocity data to enable quantitative validation of these models.

2. Experimental rig and instrumentation

The flow loop used for the present experiments was a modified version of that used by Escudier and Smith (2001) for their square-duct investigation. The square-duct consisted of ten stainless steel modules each of length 1.2m and with an internal cross section of side length 80 mm. The plane sudden expansion, for which the key dimensions are given in **Figure 1**, replaced one of the existing modules 9.6 m from the inlet connection. This arrangement provides a length of 120 hydraulic diameters for the flow to become fully developed. The duct width w throughout is 80 mm, the inlet height d is 28 mm and the step height h is 6 mm. The downstream duct height D is 40 mm. The expansion was preceded by a short (60 mm in length), smooth contraction (50 mm radius followed by 11 mm radius). The side-walls of the expansion were made of borosilicate glass to permit velocity measurements using a laser Doppler anemometer. Distributions of mean axial (U) and transverse velocity (V) were obtained from traverses at 7 axial locations (corresponding to x/h values of 0,1,2,3,4,6 and 10).

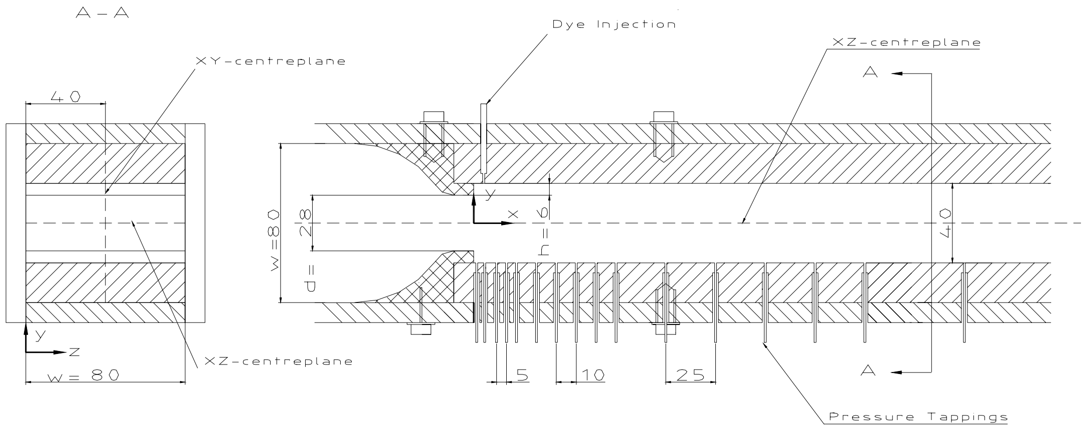

Figure 1: Plane sudden expansion geometry, dimensions in mm

A Dantec Fibreflow laser Doppler anemometer system was used for the measurements and comprised a Dantec 60X10 probe and a Dantec 55X12 beam expander in conjunction with two Dantec Burst Spectrum Analyzer signal processors (one model 57N10 the other model 57N20). The beam separation at the front lens was 51.5 mm and the lens focal length 160 mm which produces a measurement volume with principal axis of length 0.21 mm and diameter 0.02 mm. Ensemble averages were formed from not less than 9500 velocity samples.

Flow rates were measured using a Fischer and Porter electromagnetic flowmeter (model 10D1) incorporated in the flow loop upstream of the sudden expansion with the flowmeter output signal recorded via an Amplicon PS 30AT A/D converter.

All rheological measurements were carried out using a TA Instruments Rheolyst AR 1000N controlled-stress rheometer. A temperature of 25 °C was maintained for all the rheological measurements, which was also the average temperature of the fluid for the duration of the experimental runs. Temperature control of the TA rheometer is achieved via a plate that uses the Peltier effect to control the temperature of the sample to within ±0.1 °C. The rheological characterisation included measurements of effective shear viscosity, first normal stress difference N_1, the storage modulus G' and the loss modulus G''.

3. Working fluid characteristics

The working fluid used in this investigation was an aqueous solution of 0.4% w/w polyacrylamide (PAA): Separan AP273 E supplied by SNF UK limited. The solvent used was filtered tap water with 100 ppm of 40% formaldehyde solution added to prevent bacterial degradation. Approximately 0.25 gm of Timiron seeding particles were added to the fluid (total volume of fluid being 575 litres) to improve the LDA signal quality. PAA was chosen as it is highly viscoelastic, is optically transparent (thereby permitting LDA measurements) and has been used extensively in previous investigations in the same laboratory (see e.g. Escudier et al (1999) and Escudier and Smith (2001)).

PAA is generally regarded (see e.g. Walters et al (1990)) as being ‘very flexible’ in its molecular structure and it is this flexibility which gives rise to more viscoelastic characteristics compared to other water-soluble polymers such as xanthan gum and carboxymethylcellulose. The average molecular weight for the

PAA used in this study, ascertained using gel phase chromatography, was determined to be 1.94 x 10^6 g/mol with a polydispersity of 1.05.

The measured viscosity versus shear rate data for PAA is shown in **Figure 2** together with the Carreau-Yasuda model fit:

$$\mu_{CY} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{\left(1 + (\lambda_{CY}\dot{\gamma})^a\right)^{n/a}}$$

μ_0 being the zero-shear-rate viscosity, μ_∞ the infinite-shear-rate viscosity, λ_{CY} a time constant, n a power-law index and *a* a parameter introduced by Yasuda et al (1981). The parameters were determined using the fitting procedure outlined in Escudier et al (2001) and are shown in **Table 1**. The standard deviation of the model fit to the data was 1.6% and the Pearson correlation coefficient was 0.99977.

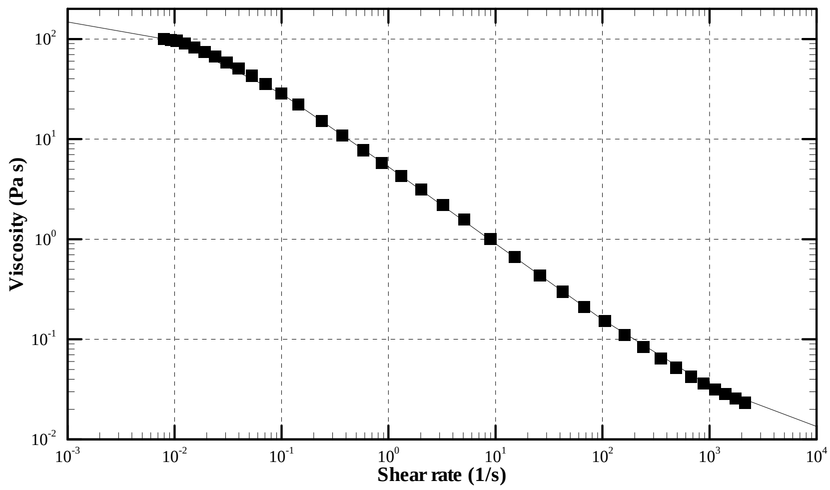

Figure 2: Viscosity versus shear rate for 0.4% polyacrylamide (including Carreau-Yasuda fit)

Table 1: Carreau-Yasuda model parameters

μ_0 (Pa s)	μ_∞ x 10^3 (Pa s)	λ_{CY} (s)	*n*	*a*
162	9.37	78.9	0.779	0.888

The first normal stress difference N_1 is the measurement most often used to classify the elasticity of a fluid. The measured variation of N_1 with shear stress is shown in **Figure 3** and between shear stress values of 20 and 40 Pa is well represented by a power-law fit:

$$N_1=0.114\tau^{2.58}$$

We note that within this shear stress range the recoverable shear $N_1 / 2\tau$, which is often taken as a measure of how elastic a liquid is, exceeds the value 5. Values this high indicate a fluid which can be classified as highly elastic (Barnes et al, 1989).

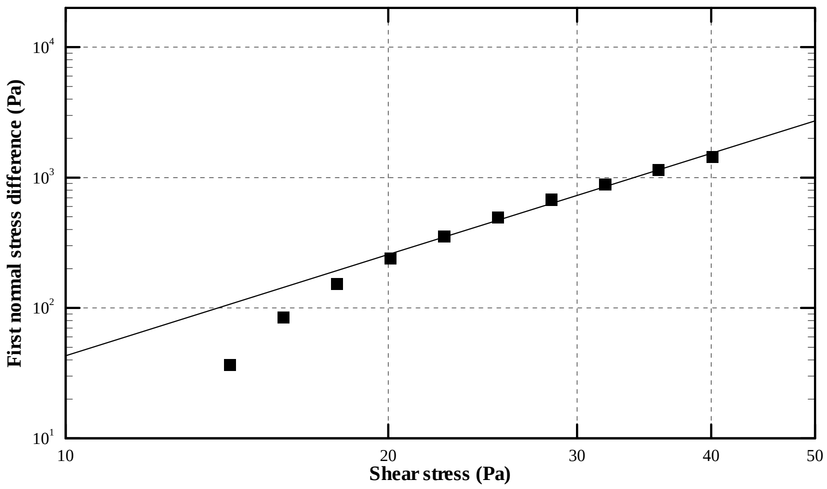

Figure 3: First normal stress difference versus shear stress for 0.4% polyacrylamide (including power law fit)

The viscoelastic properties of the PAA used were investigated further under oscillatory flow conditions. The results can be seen in **Figure 4** in terms of the storage modulus, G', and the loss modulus, G'', plotted against angular frequency. A linearity check was conducted to determine the linear viscoelastic region prior to the frequency sweep. The frequency sweep shown in **Figure 4** was performed at a shear stress of 0.5 Pa, a value well within the linear regime. Comparison with frequency-sweep data at a higher shear stress, again within the linear regime, confirmed that the viscoelastic properties observed were independent of the shear-stress value. The oscillation data confirm the high elasticity of the fluid as G' is always greater than G'' with the ratio G'/G'' increasing from 1.3 at low frequencies to a maximum of 3.2 at a frequency of 10 rad/s. At the highest frequencies the data are adversely affected by the effects of inertia and should be viewed with caution.

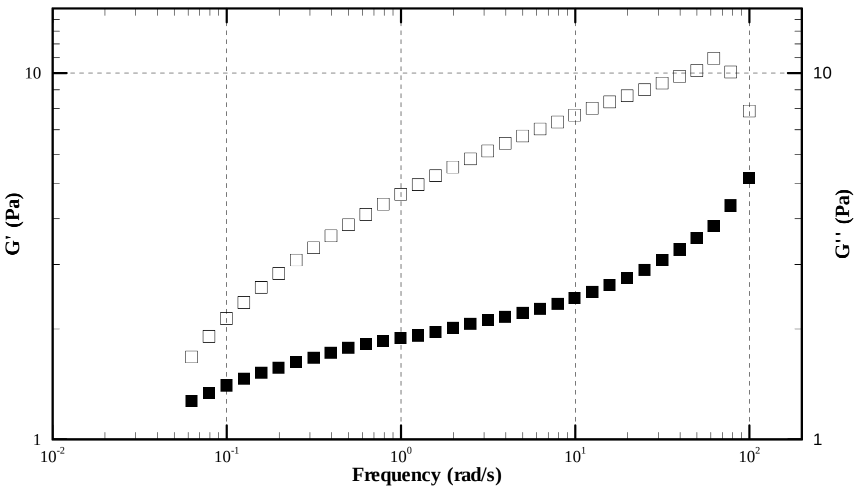

Figure 4: Storage (G′ ☐) and loss (G′′ ■) moduli versus angular frequency for 0.4% polyacrylamide

4. Estimation of Reynolds number

Many workers (see for example Townsend and Walters (1994)) do not attempt to define a Reynolds number for shear-thinning liquids because no single value for the viscosity completely characterises the fluid. In spite of this difficulty we have quantified the relative importance of inertial to viscous forces as follows. The bulk velocity determined from the flowmeter (U_B= 0.132 m/s =Q/A where A = inlet area at the expansion plane = wd) was used as a characteristic velocity. The step height (h=0.006 m) was taken as the appropriate length scale as is customary in backward-facing step flows (Eaton and Johnston (1981)). The density of the fluid was constant and essentially that of the solvent, water. We have estimated the shear rate at the wall (assuming a linear fit between the velocity at the first measurement point and the wall) at the inlet to the sudden expansion (x/h=0) and then obtained a viscosity by substituting this into the Carreau-Yasuda model:

$$\dot{\gamma}_c = \left.\frac{dV}{dy}\right|_{Wall} = 97.5 s^{-1}$$

which gives a viscosity of μ_c=0.16 Pa.s and hence a Reynolds number of

$$\mathrm{Re} = \frac{\rho U_B h}{\mu_c} \approx 5.$$

We note also that the recoverable shear at this location (i.e $N_1/2\tau$) is approximately 2.5.

5. Discussion

5.1 Inlet velocity profile

The transverse variation of the inlet velocity profile on the duct centreline, i.e. the velocity profile at x/h =0 in the XY-centreplane, is shown in **Figure 5**.

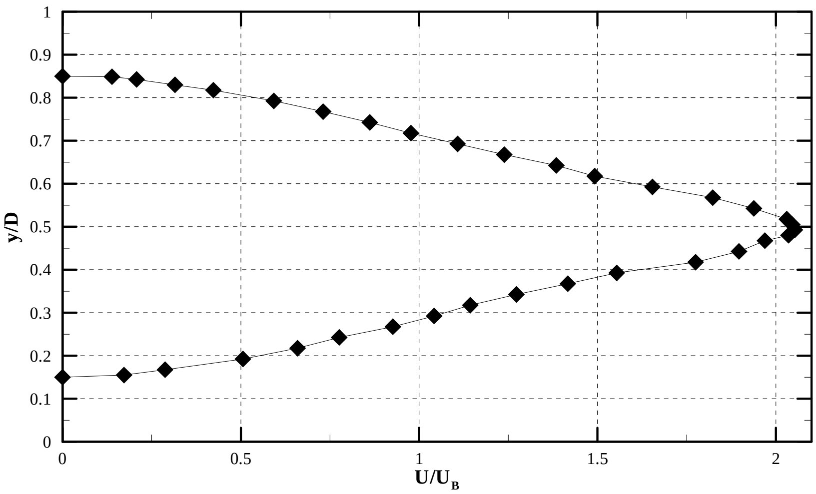

Figure 5: Transverse variation of inlet velocity profile in the XY centreplane

The shape of the velocity profile is a consequence of the smooth contraction immediately preceding the expansion. For a Newtonian fluid the profile would be almost uniform. Although the velocity profile upstream of the smooth contraction was not measured it is reasonable to assume that it is fully developed as there is a development length of some 120 hydraulic diameters. It is apparent that the contraction has accelerated the central core region of the flow significantly and produced an almost triangular profile which is symmetrical with respect to the XZ-centreplane (y/D=0.5). However, numerical integration of this velocity profile led to a bulk velocity much higher than that obtained from the flowmeter suggesting that the flow was far from two-dimensional. To investigate this further we measured the spanwise variation of the inlet profile, i.e. in XZ planes, with the results seen in **Figure 6**.

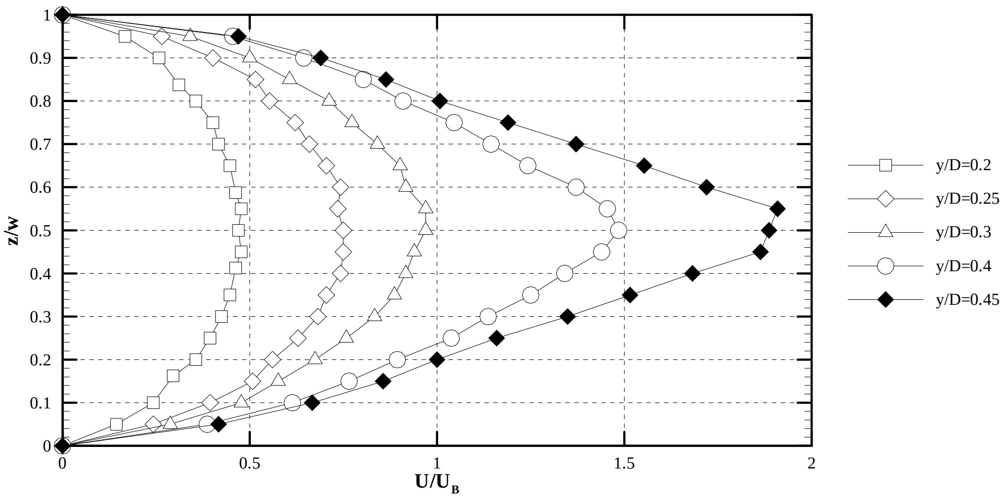

Figure 6: Spanwise variation of inlet velocity profile in XZ planes

Symmetry about the XY-centreplane (z/w=0.5) is evident, but shows that the flow is far from two-dimensional with a "parabolic variation" of velocity at transverse heights less than y/D=0.3 whereas nearer the XZ-centreplane (i.e. y/D =0.4 and 0.45) the profile takes on a more triangular shape. It is also observable (see **Figure 6**) that for increasing y/D values the shear rate at the wall increases. The increased normal stress difference associated with this higher shear rate, combined with the presence of the side-walls, would have the effect of 'pushing' the flow towards the XY-centreplane (z/w=0.5). This variation of normal stress difference with transverse distance is the most likely cause of the unusual inlet velocity profile.

5.2 Mean axial and transverse velocity profiles downstream of sudden expansion

Axial velocity profiles downstream of the sudden expansion in the XY-centreplane are shown in **Figure 7**. The symmetry was checked and the results (×), included in **Figure 7** for x/h=0 and 2, confirm that the velocity profiles are symmetrical above and below the XZ-centreplane (y/D=0.5). Due to this symmetry only half profiles are presented for the remaining profiles (x/h=1,3,4,6 and 10).

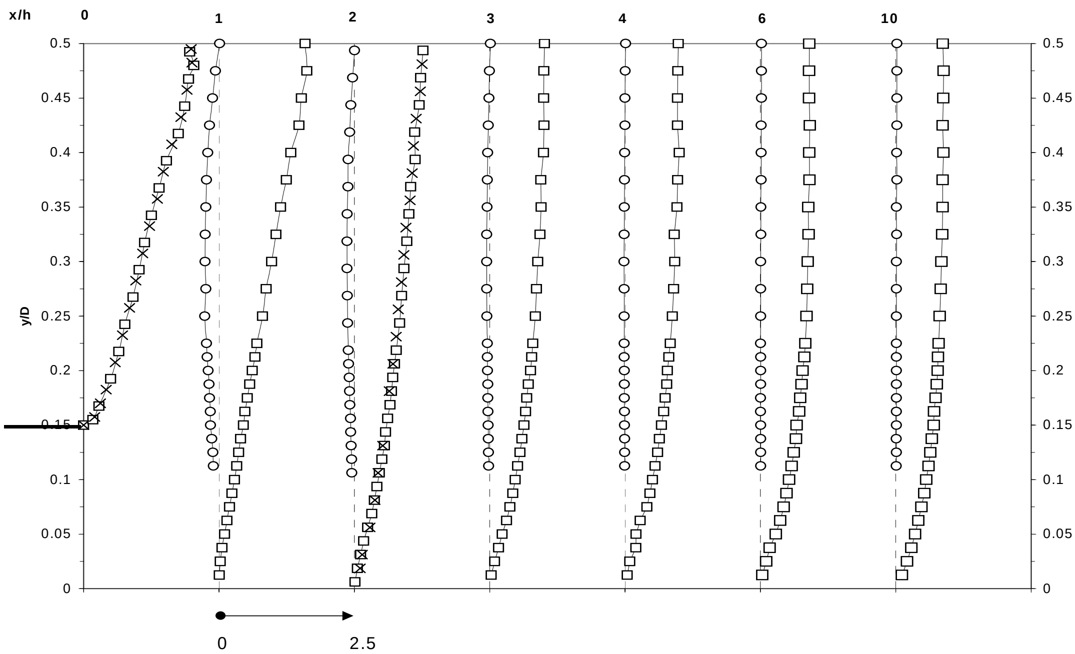

Figure 7: Mean axial (U/U_B □) and transverse (V/U_B ○) velocity profiles in the XY-centreplane

These results show that the plane sudden expansion for this flow essentially acts as a double backward facing step. For Newtonian flows this symmetry is not always the case and asymmetry is seen beyond critical Reynolds numbers when the expansion ratio (R=D/d) exceeds 1.5. It is for this reason that in the current study the expansion ratio was set at 1.43.

Despite the complicated inlet profile the flow field shows the same characteristics as similar flows reported previously, see for example Townsend and Walters (1994). The recirculation region seen in Newtonian flows is absent and the entire velocity field is in the positive streamwise direction. In the near vicinity of the wall the velocities are very low: less than 2% of U_B. It appears that as the fluid enters the expansion the elastic stresses in the fluid relax and this mechanism allows expansion in the transverse flow direction and hence the suppression of recirculation. This process is analogous to the well-known phenomenon of die-swell, which occurs when a viscoelastic liquid is extruded from a die or flows from the exit of a tube, see Barnes et al (1989). Streamlines are not presented due to the lack of two-dimensionality of the flow about the XY-centreplane, illustrated in **Table 2**, which shows the apparent flowrate for each axial profile determined from numerical integration of the mean axial velocity profile. **Table 2** indicates that the effect of the smooth contraction is to increase the apparent flowrate along the XY-centreplane of the duct. Directly after the expansion the apparent flowrate increases before decreasing with downstream distance suggesting that not only is the flow expanding in the transverse direction but also in the spanwise. The near constant apparent flowrate (±3%) after x/h=3 shows that this process takes place in the near vicinity of the expansion.

Table 2: Apparent flow rates at various axial locations in the XY centreplane

	Flowmeter	x/h=0	x/h=1	x/h=2	x/h=3	x/h=4	x/h=6	x/h=10
Q (m^3/h)	1.068	1.201	1.284	1.200	1.142	1.160	1.119	1.112

The transverse, V, velocity distribution (i.e. in the y direction) is also included in **Figure 7**. The maximum negative transverse velocity occurred immediately after the expansion, at x/h=1, and was approximately $0.3U_B$. After this location the magnitudes of the transverse velocities diminish before attaining a value of zero at downstream distances greater than three step heights.

6. Conclusions

Results have been reported of an experimental investigation of the laminar flow ($Re \approx 5$) of a viscoelastic liquid flowing through a plane sudden expansion immediately preceded by a smooth contraction.

The inlet velocity profile produced by the smooth contraction is intriguing. In the transverse direction the velocity profile has an accelerated central core and is almost triangular in shape whilst in the spanwise direction it is parabolic in shape close to the step but more triangular in shape towards the XZ-centreplane. The peculiar three dimensional velocity profile is a consequence of variations in the magnitude of the first normal stress difference.

The flow field downstream of the expansion is in the positive streamwise direction and devoid of recirculation, this being a consequence of the viscoelasticity of the fluid damping out vortex activity.

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