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Turbulent flow of non-Newtonian liquids over a backward-facing step Part I. A thixotropic and shear-thinning liquid

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Abstract

The results are reported of an experimental investigation of turbulent flow of a 1.5% Laponite solution, a shear thinning and thixotropic liquid, through a plane sudden expansion of expansion ratio $R = 1.43$ and aspect ratio $A = 13.3$. Two Newtonian fluid flows are also reported for comparative purposes. Apart from a slight effect on the reattachment length, caused by variations in the maximum turbulence intensity at separation, no major differences are found between the mean and turbulent flow characteristics of the Newtonian and Laponite fluid flows.

Keywords: Thixotropic; Shear thinning; Turbulent; Backward-facing step

1. Introduction

The objective of the present study, reported in two parts, is to examine the influence non-Newtonian fluid rheology has on the turbulent separation and reattachment process behind a backward-facing step. To the authors' knowledge there have been no previous investigations in this field. In this paper (Part I) we discuss the flow of a shear thinning, thixotropic liquid. In Part II [1], we discuss an investigation of the flow of various concentrations of polyacrylamide solutions, which are viscoelastic and shear thinning. The same geometry was used in both parts of the study.

The reattachment of a turbulent shear layer is of fundamental importance in numerous engineering applications. The plane sudden expansion is, for example, relevant to such diverse applications as fluidic devices, heat exchangers, mixing equipment and air-conditioning ducts. For Newtonian fluids the first authors to investigate plane sudden expansions were Abbott and Kline [2]. They concluded that for expansion ratios ($R = D/d$, D being the downstream channel height and d the inlet height) greater than 1.5 the flow field was asymmetric but that below this value the flow geometry acts essentially as

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Nomenclature

A	aspect ratio w/h
d	duct height at inlet (m)
D	downstream duct height (m)
h	step height (m)
K	consistency parameter (Pa s^n)
n	power-law index
$\dot{Q}_A$	flowrate determined by numerical integration (m^3/s)
$\dot{Q}_F$	flowrate reading from flowmeter (m^3/s)
$\dot{Q}_R$	recirculating flowrate determined by numerical integration (m^3/s)
R	expansion ratio, D/d
Re	Reynolds number ($\equiv \rho h U_B/\mu_{SEP}$)
u'	streamwise turbulence intensity (m/s)
u'_0	freestream turbulence intensity (m/s)
u'_{SEP}	maximum streamwise turbulence intensity at separation (m/s)
$\overline{uv}$	Reynolds shear stress (m^2/s^2)
U	mean streamwise velocity (m/s)
U_0	freestream velocity (m/s)
U_B	bulk mean velocity $\dot{Q}_F/wd$ (m/s)
U_{Rmax}	maximum recirculating streamwise velocity (m/s)
V	mean transverse velocity (m/s)
v'	transverse turbulence intensity (m/s)
w	channel width (also duct height upstream of smooth contraction) (m)
x	streamwise distance from expansion (m)
x_R	reattachment length (m)
X_R	non-dimensional reattachment length (x_R/h)
y	transverse distance from downstream duct floor (m)

Greek letters

$\dot{\gamma}$	shear rate (s^{-1})
δ	initial boundary layer thickness (m)
μ_{SEP}	Hershel–Bulkley viscosity corresponding to shear rate at inlet (Pa s)
τ	shear stress (Pa)
τ_Y	yield stress (Pa)

Subscripts

0	freestream
max	maximum

a double backward-facing step and the flowfield is symmetric. Since that time a considerable literature has developed devoted to the subject of turbulent flow of a Newtonian fluid over a backward-facing step whereas investigations for the asymmetric ($R > 1.5$) configuration are much more limited [3]. Apart from its practical relevance, the backward-facing step is an ideal test case for the development and validation of turbulence models. In the current study, the expansion ratio is 1.43.

The first 20 years of backward-facing step research for Newtonian fluid flows is documented succinctly in the review of Eaton and Johnston [4], which encompasses data from more than 20 authors and shows that, dependent upon initial and boundary conditions, the reattachment zone ranges between 4.9 and 8.2 step heights in length. Four principal independent parameters are suggested as having a significant effect on the reattachment process.

1. The state (laminar/turbulent) of the separation boundary layer: the downstream flow is independent of the momentum thickness Reynolds number when the boundary layer is fully turbulent.
2. The initial boundary layer thickness (δ): the reattachment length is found to decrease with increasing δ.
3. Freestream turbulence: Eaton and Johnston [4] suggest that high levels of freestream turbulence decrease the reattachment length.
4. The streamwise pressure gradient, which is controlled in part by the overall system geometry: reattachment length generally increases with increasing expansion ratio.
5. In addition, de Brederode and Bradshaw [5] show that the effect of the aspect ratio ($A = w/h =$ channel width/step height) is negligible for $A > 10$.

There have been a number of studies of flow over a backward-facing step since Eaton and Johnston's [4] review. The investigation of Adams et al. [6] concentrated on the state of the boundary layer at separation, its thickness, the effect of Reynolds number and the expansion ratio. It was concluded that the primary variable affecting the reattachment location was the boundary layer state (laminar/turbulent) rather than its thickness. They showed that once the boundary layer is fully turbulent, increases in δ lead to only a slight decrease in the distance to reattachment. Westphal and Johnston [7] demonstrated that distributions of pressure coefficient, skin friction coefficient and forward-flow fraction plotted against $(x - X_R)/X_R$ appear universal for two-dimensional reattachment, independent of initial conditions and step height for thin separating boundary layers.

Isomoto and Honami [8] investigated the influence on reattachment length of inlet turbulence levels within the boundary layer manipulated using two-dimensional rods and cavities upstream of the step and also of grid-generated freestream turbulence. They concluded that the intensity levels of freestream turbulence had little effect but that the maximum turbulence intensity near the wall at the location of separation had a major influence on the reattachment length. For example, an increase of $0.02U_B$ in the turbulence intensity near the wall was shown to decrease the reattachment length by two step heights.

Although many naturally occurring fluids, and the majority of synthetic fluids such as those encountered in the food, processing and chemical industries, are non-Newtonian in character, we are unaware of any papers on the turbulent flow of non-Newtonian liquids through either plane sudden expansions or over backward-facing steps. Viscoelasticity and pseudoplasticity inevitably add further complexity to what is an already highly turbulent and complex flow. However, limited progress has been made in understanding turbulent recirculating and reattaching non-Newtonian fluid flow in the axisymmetric sudden expansion configuration. Pak et al. [9] used flow visualisation to investigate the flow of two non-Newtonian liquids through an axisymmetric sudden expansion: a purely viscous shear-thinning liquid, Carbopol, (and viscoelastic polyacrylamide solutions). The reattachment lengths for Carbopol were found to be essentially the same as for water. We shall discuss the influence the influence of viscoelasticity for the present geometry in Part II.

Castro and Pinho [10], Pereira and Pinho [11,12] have also investigated the flow of a series of non-Newtonian liquids with fully developed inlet velocity profiles through an axisymmetric sudden expansion of expansion ratio 1.54. Castro and Pinho [10] used Tylose solutions, which are moderately shear thinning ($n \approx 0.7$) and practically inelastic, and noted only small changes in the mean flow but significant reductions (up to 30%) in the normal Reynolds stresses. Pereira and Pinho [11] used xanthan gum as their working fluid, as did Escudier and Smith [13] in a companion study of identical expansion ratio but with a smooth contraction preceding the expansion which produced a uniform inlet profile with low turbulence intensity. Escudier and Smith observed no change in the mean flow but considerably reduced levels (up to 20%) of turbulent kinetic energy. In contrast Pereira and Pinho [11] reported a reduction in the reattachment length of more than 20% relative to a Newtonian fluid flow with a similar *Re*. This difference was attributed to the dominating role of inlet turbulence (much as Isomoto and Honami [8] observed for Newtonian fluid flow over a backward-facing step) with higher axial turbulence intensity and lower levels of turbulence anisotropy at inlet in the study of Pereira and Pinho [11] associated with their fully developed inlet velocity profile. Pereira and Pinho [12] concluded that both for the mean flow and the turbulent structure, the flow of a 1% Laponite solution, a shear thinning, thixotropic, essentially inelastic fluid, was little different to that of water.

For Newtonian fluids it is well known [2] that plane sudden expansions with $R > 1.5$ produce asymmetric flows with two recirculation regions of unequal length but, below this critical R-value, the flow acts essentially as a double backward-facing step. In the current study, the expansion ratio is 1.43 with the intention of generating a symmetric flow, assuming the $R > 1.5$ criterion applies to non-Newtonian fluids. A laser Doppler anemometer was used to measure mean and RMS streamwise velocities, U and u', as well as the transverse mean and RMS velocities, V and v', and the Reynolds shear stress $\overline{uv}$. A 1.5% Laponite solution was chosen as the working fluid and two Newtonian fluid flows are also reported for comparative purposes.

2. Experimental rig and instrumentation

The flow loop used for the present experiments was a modified version of that used by Escudier and Smith [14] for their square duct investigation. The square duct consisted of 10 stainless steel modules each of length 1.2 m with an internal cross section of side length 80 mm. The double backward-facing step, for which the key dimensions are given in Fig. 1, was installed in a replacement module 9.6 m from the inlet connection. The duct width w remained 80 mm throughout, the inlet height d is 28 mm, the step height h is 6 mm and the downstream duct height D is 40 mm. These dimensions produce an expansion ratio $R = D/d = 1.43$ and an aspect ratio $A = w/h = 13.3$. By choosing $R < 1.5$ it was anticipated that the flow would be symmetrical if the Abbott and Kline [2] criterion also applies to non-Newtonian fluid flow. Similarly, a value of $A > 10$ was chosen to satisfy de Brederode and Bradshaw's [5] criterion for minimising end effects and three dimensionality.

The expansion was preceded by a short (53.5 mm in length), smooth contraction (40 mm concave radius followed by 20 mm convex radius) which for the higher Reynolds number flows led to a distribution of velocity at the plane of the sudden expansion which was practically uniform and of low turbulence

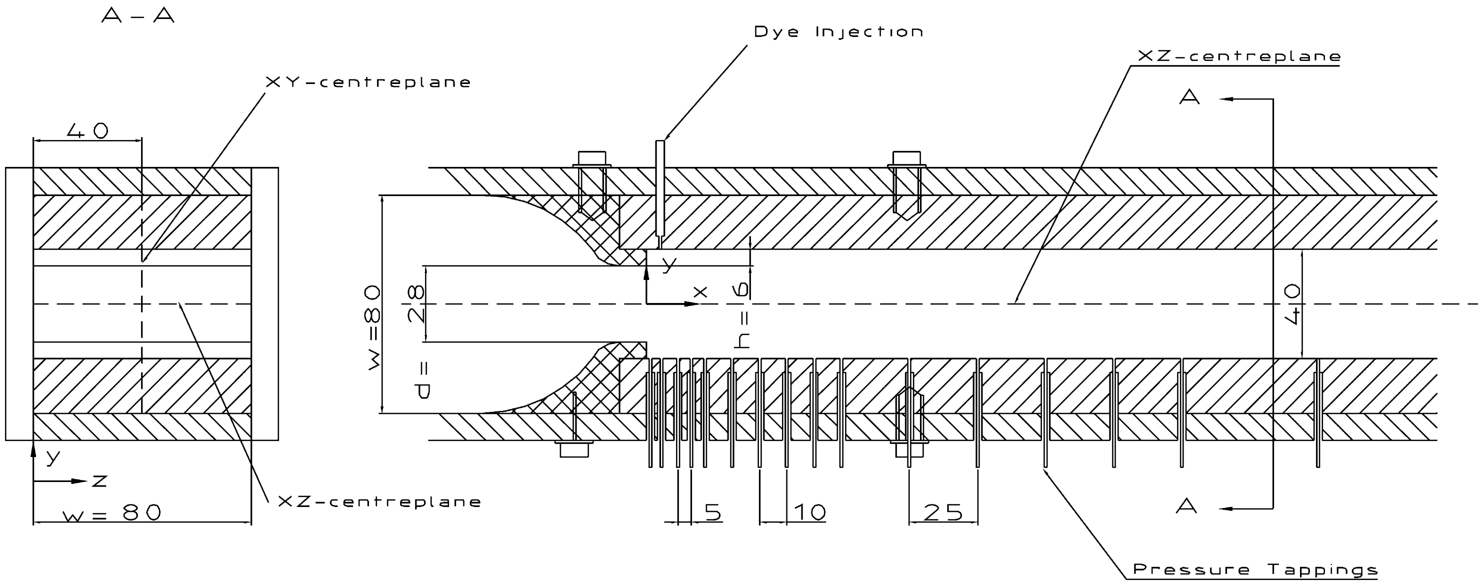

Fig. 1. Backward-facing step geometry, dimensions in millimetre.

intensity. At lower Reynolds numbers, the contraction was found to be less effective and produced a marginally thicker initial boundary layer with increased turbulence intensity near the wall. The side-walls of the expansion were made of borosilicate glass to permit velocity measurements using a laser Doppler anemometer. Distributions of mean velocity and turbulence structure were obtained along the *XY*-centreplane of the duct from traverses at 10 streamwise locations corresponding to x/h values of 0, 1, 2, 3, 4, 5, 6, 9, 12 and 18. The region below $y/h = 0.8$ was inaccessible to the laser Doppler anemometer (LDA) beams in the *YZ*-plane and so no transverse turbulence intensities or Reynolds shear stresses could be reported below this level.

A Dantec fibreflow laser Doppler anemometer system was used for the velocity and turbulence measurements and comprised a Dantec 60X10 probe and a Dantec 55X12 beam expander in conjunction with two Dantec Burst Spectrum Analyzer signal processors (one model 57N10, the other model 57N20). The beam separation at the front lens was 51.5 mm and the lens focal length 160 mm (corresponding to an included half angle of 9.14°) which produces a measurement volume with principal axis of length 0.21 mm and diameter 20 μm. In view of the small diameter of the measuring volume, no correction was applied for the effect of velocity-gradient broadening. The streamwise and transverse velocity values were collected in coincidence to enable the Reynolds shear stress values to be estimated. As recommended by Tropea [15], transit-time weighting was used to correct the velocity measurements for the effects of velocity bias. Nominally, 10,000 velocity samples were collected which resulted in a maximum relative statistical error, for a 95% confidence interval, of approximately 0.5% in the mean velocity and 1.4% in the turbulence intensity [16]. The total uncertainty in the mean velocity is estimated to be in the range 3–4%.

As shown in Fig. 1, 19 pressure tappings of 1 mm diameter were provided along the *XY*-centreplane of the expansion to allow the wall pressure variation to be measured. The tappings were connected to 2 mm ID clear vinyl tubing, filled with deionised water, linking each in turn via a series of valves to a Validyne differential pressure transducer (model DP15-26). Flow rates were measured using a Fischer and Porter electromagnetic flowmeter (model 10D1) incorporated in the flow loop upstream of the sudden expansion with the flowmeter output signal recorded via an Amplicon PS 30AT A/D converter.

All rheological measurements were carried out using a TA Instruments Rheolyst AR 1000N controlled stress rheometer. A temperature of 20 °C was maintained for the rheological measurements, which was also the average temperature of the fluids for the duration of the experimental runs. Control of the temperature of the sample to within ±0.1 °C is achieved in the rheometer via a plate using the Peltier effect.

3. Rheology of working fluid

The non-Newtonian fluid chosen for investigation was a 1.5 wt.% solution (in water) of Laponite, a complex fluid exhibiting thixotropy, shear thinning, a yield stress and viscoelasticity. The Laponite used in this study is a synthetic hectorite clay manufactured by Laporte Industries with a structure similar to that of sodium montmorillonite, a principal constituent of Wymoning Bentonite. The chemical structure of Laponite is detailed in Escudier and Presti [17].

Measurements of viscometric properties were obtained using a consistent procedure to produce a consistent shear history of the fluid. The fluid was circulated in the flow loop for 30 min at a flowrate of circa 40 m^3/h before the rheological measurements were conducted. The fluid was then pre-sheared in

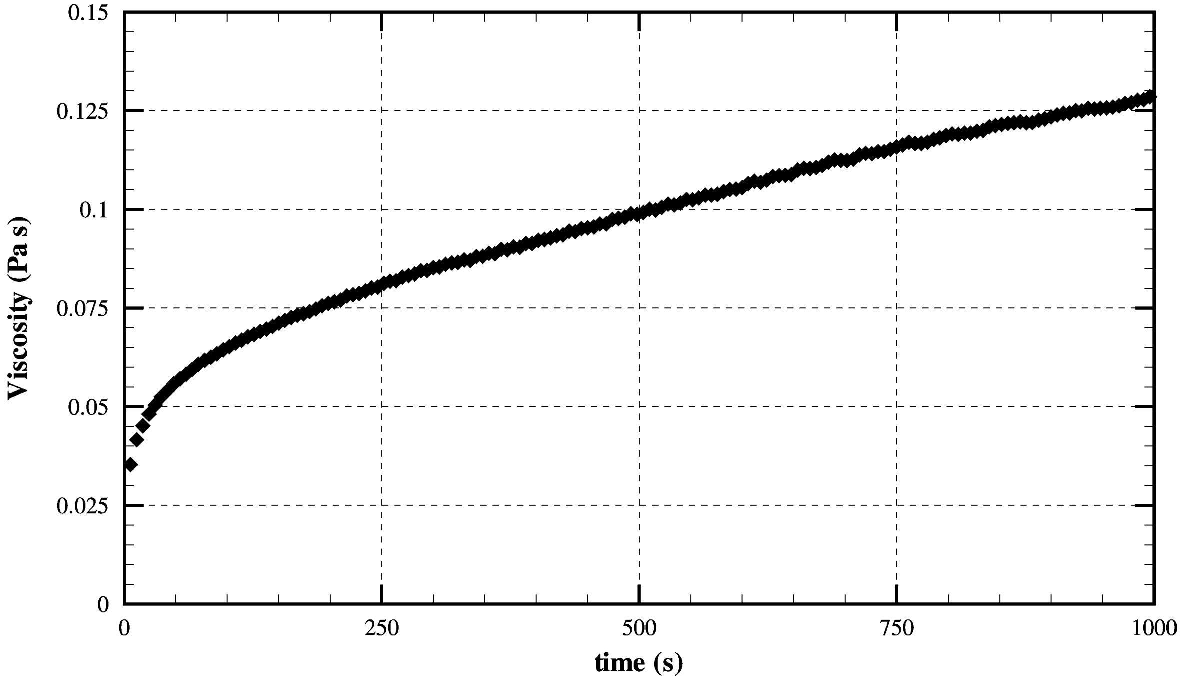

Fig. 2. Thixotropic behaviour of 1.5% Laponite sheared at 7 Pa after pre-shearing at 12 Pa.

the rheometer for 1 h at a shear stress of 8 Pa. The data were then obtained using a 'peak hold' procedure whereby the fluid was sheared at a given shear stress and the shear rate monitored over time. Each procedure took of the order of 3600 s to reach an equilibrium value. The fluid was deemed to be in equilibrium when the change in shear rate over a period of 600 s was less than 0.5%. Due to the large time required to obtain rheological data a solvent trap was always used to minimise sample evaporation. The thixotropic nature of Laponite is clearly evident from data presented in Fig. 2 which shows viscosity as a function of time for a Laponite sample sheared at 7 Pa after a pre-shear at 12 Pa for 3600 s. Fig. 3 shows the variation of shear stress versus shear rate from data also obtained using the above procedure. The fitted curve corresponds to the Herschel–Bulkley model:

$$\tau = \tau_Y + K\dot{\gamma}^n$$

where $\tau_Y = 3.43$ Pa is the 'yield stress', $K = 0.288$ Pa s^n the consistency parameter and $n = 0.518$ the power-law index. The existence of a 'true' yield stress has been widely debated [18]. The term apparent yield stress is probably more appropriate and the value of τ_Y should be regarded as a fitting parameter strongly influenced by the low shear rate data. The first normal stress difference was also measured and showed that the Laponite was essentially inelastic. The Newtonian fluid was tap water.

4. Results and discussion

Only half profiles are reported since symmetry checks confirmed that each flow was symmetrical about the *XZ*-centreplane. To investigate flow two dimensionality, each of the mean streamwise velocity profiles,

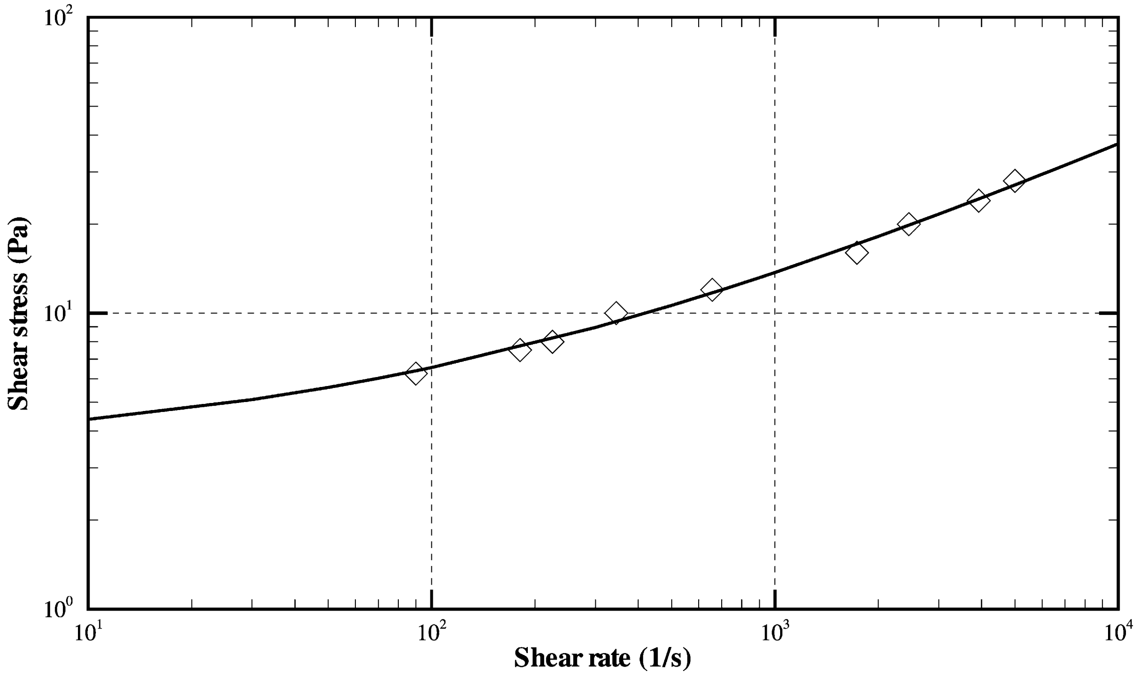

Fig. 3. Shear stress vs. shear rate for 1.5% Laponite (including Herschel–Bulkley fit).

shown in Fig. 4, was integrated to reveal an apparent flowrate and this value compared to the flowrate indicated by the flowmeter. The agreement for all three flows was better than 5%.

Fig. 4 reveals that mean streamwise velocity profiles for the two Newtonian fluids ($Re = 14{,}000$ and 40,000) are extremely similar to each other and to the Laponite profiles. The slightly thicker initial boundary layer thickness δ (see Table 1) of the Laponite solution is associated with its lower Re ($\equiv \rho h U_B/\mu_{SEP}$) of 8700. The viscosity at separation from the step μ_{SEP} was based on the Herschel–Bulkley viscosity corresponding to the shear rate at separation assuming a linear fit between the first measuring point (0.5 mm from wall) and the no-slip condition at the wall. A decrease in δ is observed with increasing Reynolds number for all three flows. The magnitude of the boundary layer thickness is a consequence of the smooth contraction immediately preceding the expansion. The boundary layer thicknesses for all three flows are small and as such may be affected by mean gradient broadening. The effect was minimised by having a very small measuring volume and by traversing in the direction of the smallest dimension (i.e. the diameter, 20 μm). What we observe is that in addition to this boundary layer thickness difference, the reattachment length is marginally shorter, $X_R = 6$ compared to the Newtonian values of 6.3 and 6.5 step heights. The streamline patterns for water ($Re = 14{,}000$) and the 1.5% Laponite flow are shown in Fig. 5(a) and (b) respectively and confirm the essential similarity between the flows. The two Newtonian fluid flows, together with another of lower Reynolds number, will be discussed in more depth in Part II.

The streamwise and transverse turbulence intensities for the three flows are shown in Fig. 6 and reveal the turbulence structure behind the step. (Note that below $y/h = 0.8$ no transverse turbulence intensities or Reynolds shear stress values are reported due to the inaccessibility of the transverse LDA beams in this region.) In agreement with the mean flow the differences between the Newtonian and the Laponite

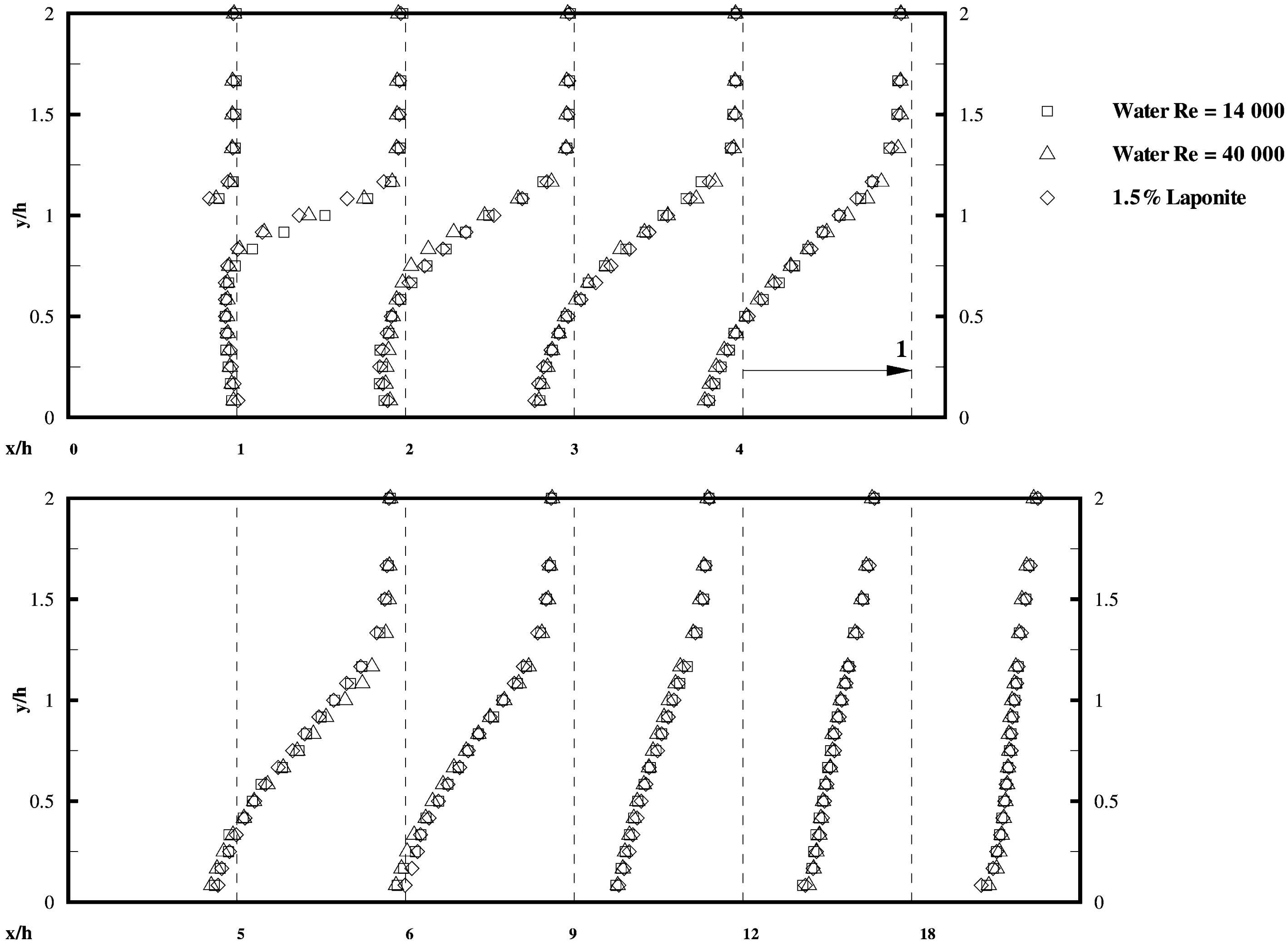

Fig. 4. Mean streamwise velocity (U/U_B) profiles in XY-centreplane.

Table 1
Representative mean flow and turbulence properties

Fluid	Re ($\equiv rhU_{\mathrm{B}}/\mu_{\mathrm{SEP}}$)	δ/h	U_{B} (m/s)	U_0 (m/s)	$u'_{\mathrm{SEP}}/U_{\mathrm{B}}$	u'_0/U_{B}	$U_{\mathrm{Rmax}}/U_{\mathrm{B}}$	$u'_{\mathrm{max}}/U_{\mathrm{B}}$	$v'_{\mathrm{max}}/U_{\mathrm{B}}$	$\overline{uv}_{\mathrm{max}}/U_{\mathrm{B}}^2$	$\dot{Q}_{\mathrm{R}}/\dot{Q}_{\mathrm{A}}$ (%)	X_{R} (x_{R}/h)
Water	14,100	0.20	2.35	2.36	0.117	0.0234	−0.204	0.223	0.137	−0.0140	3.36	6.33
Water	40,000	0.15	6.67	6.54	0.090	0.0192	−0.225	0.235	0.153	−0.0176	3.18	6.50
1.5% Laponite	8,700	0.25	5.18	5.10	0.129	0.0193	−0.234	0.218	0.140	−0.0142	3.45	6.00

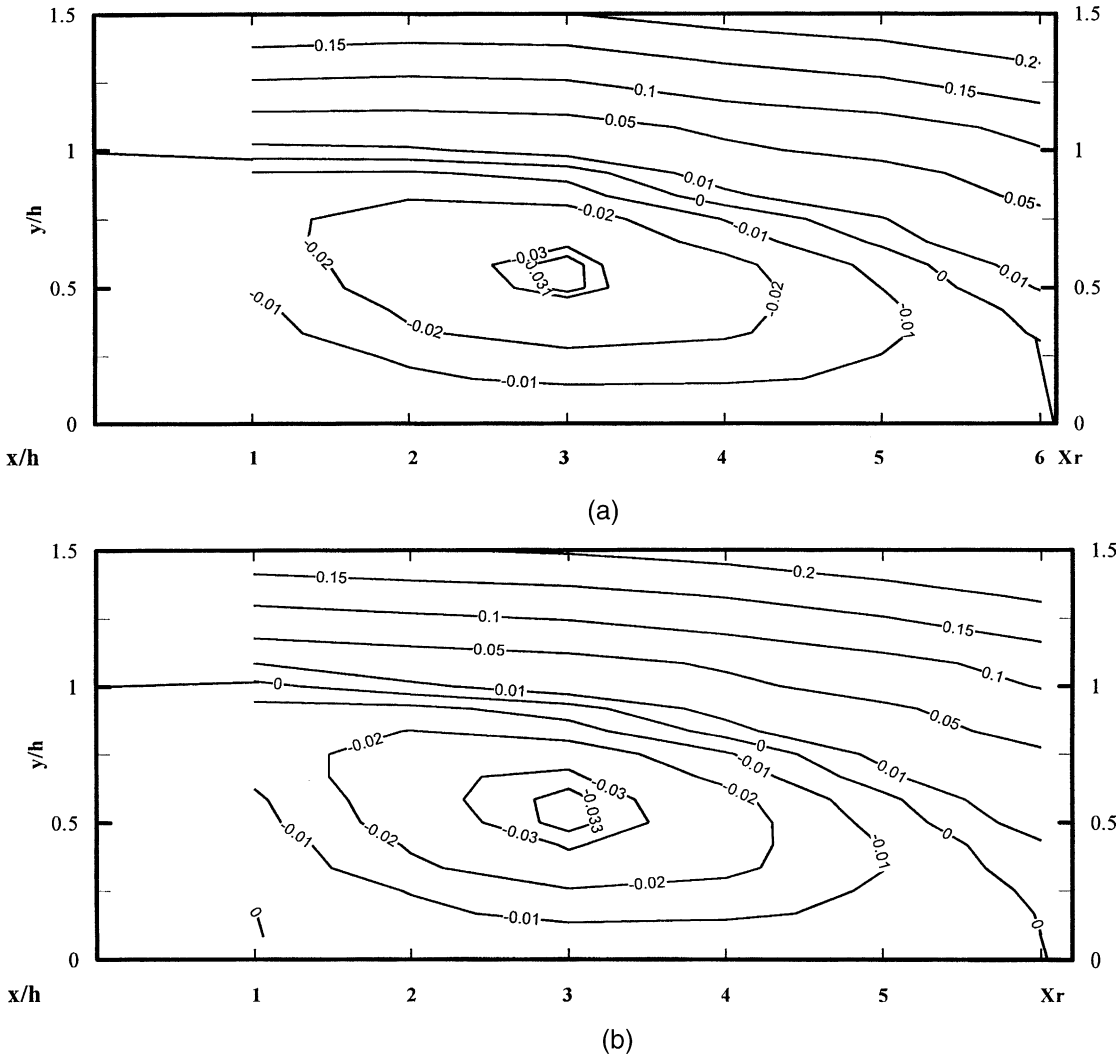

Fig. 5. (a) Streamline pattern for water $Re = 14{,}000$. (b) Streamline pattern for 1.5% Laponite $Re = 8700$.

fluid flows are small. At inlet the lower Re of the Laponite solution leads to an increase in the maximum streamwise intensity at separation which is in agreement with the trend present in the Newtonian data with the $Re = 14{,}000$ fluid flow having a higher maximum intensity at separation than the $Re = 40{,}000$ fluid flow. The reattachment lengths are inversely proportional to the maximum turbulence intensity at separation much as Isomoto and Honami [8] observed and it would seem likely that the differences in the reattachment lengths for the three flows can be attributed to this effect, additional data presented in Part II is in agreement with this hypothesis.

The streamwise intensity profiles are identical in shape for the three fluid flows and the peak values follow the same trajectory. A minor discrepancy is observed for water flow at $Re = 40{,}000$ between $4 < x/h < 12$, where both streamwise and transverse intensity components are slightly increased

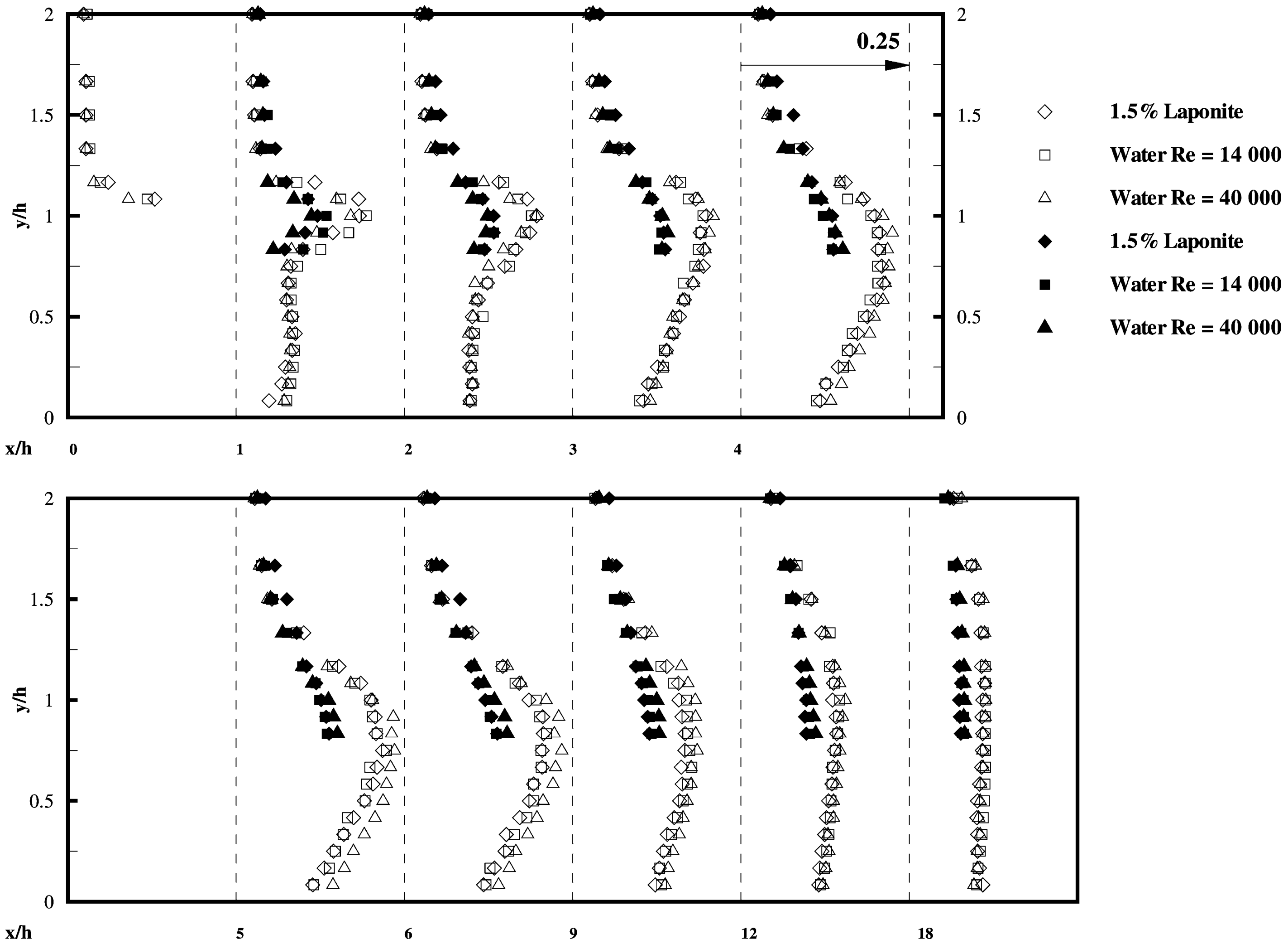

Fig. 6. Streamwise (u'/U_B, closed symbols) and transverse (v'/U_B, open symbols) turbulence intensity profiles.

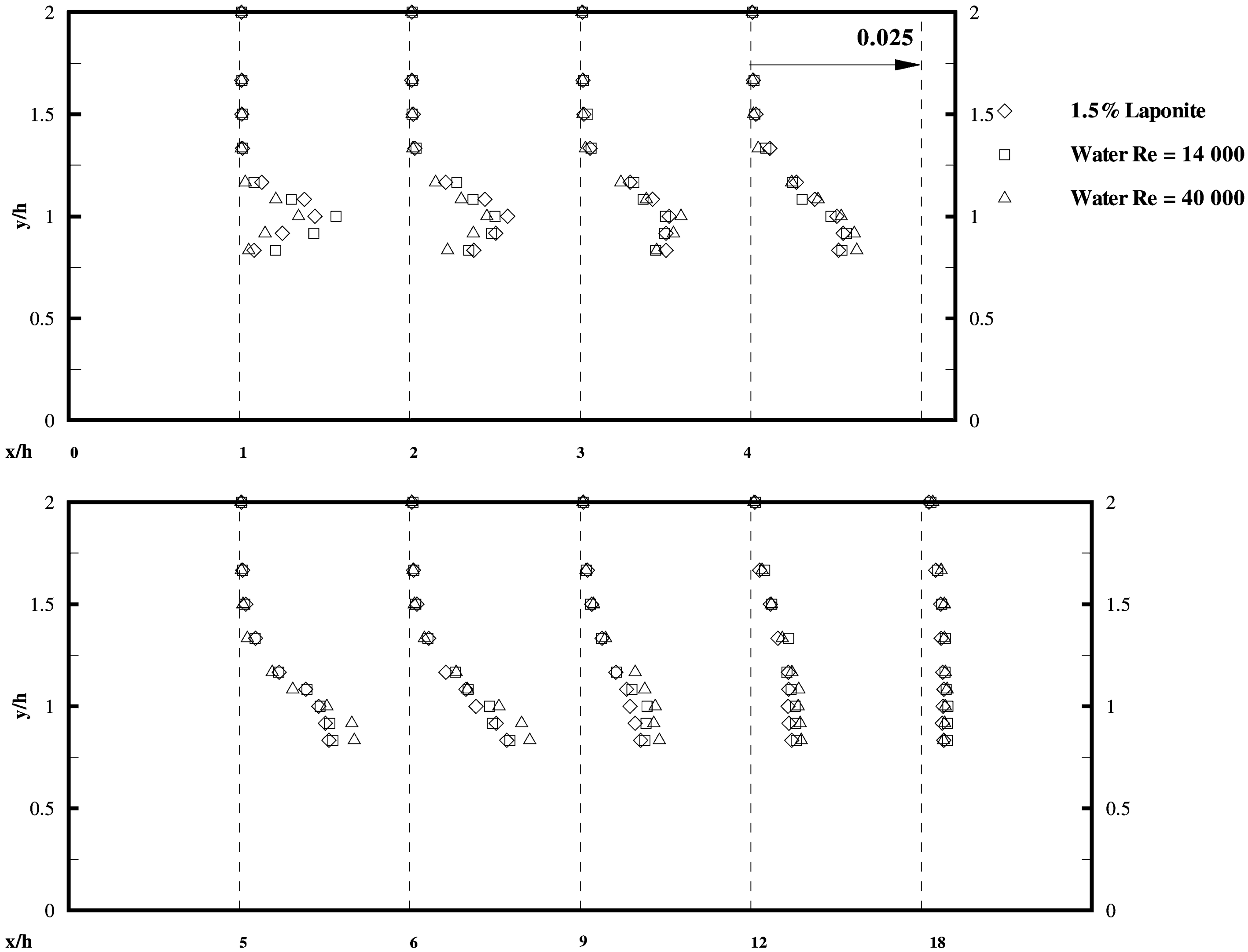

Fig. 7. Reynolds shear stress ($-uv/U_B^2$) profiles.

compared to the other two flows and this is perhaps a consequence of the higher Reynolds number of this flow. In the shear layer the turbulence is always anisotropic with $u' > v'$. It is only above the shear layer in the high velocity core that the turbulence approaches isotropy, with the exception of the Laponite flow where in this region $v' > u'$ and this appears to be the only difference between the water and Laponite flows. As the Laponite flow progresses downstream this behaviour diminishes until $x/h = 18$, where the transverse intensities are equal for Laponite and water but significantly smaller than the streamwise component with $u'/v' \approx 1.4$ for all three flows.

The maximum turbulence intensities, $u'_{\mathrm{max}}/U_{\mathrm{B}}$, etc. are in close agreement for all three flows (see Table 1) with the ratio $u'_{\mathrm{max}}/v'_{\mathrm{max}} \approx 1.6$. The Reynolds shear stress profiles of Fig. 7 are again very similar for all three flows. In the high velocity core the Reynolds shear stress is essentially zero in value. The maximum value (see Table 1) is identical for Laponite and water at $Re = 14{,}000$ but the higher *Re* Newtonian fluid flow has a maximum value about 26% higher, consistent with the larger streamwise and transverse turbulence intensities.

5. Conclusions

Despite its complex rheological nature (combining thixotropy and shear thinning), the turbulent flow of a 1.5 wt.% Laponite solution behind a backward-facing step is little different to that of water. Minor differences occur in the transverse turbulence intensity in the high-velocity core but these changes have a negligible effect on both the turbulence structure in the shear layer and the mean flow characteristics. As is confirmed by additional data in Part II, slight differences in the reattachment lengths for the three flows are attributable to differences in the maximum turbulence intensity at separation which are, in turn, related to the Reynolds number of the flow through the smooth contraction.

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