Dr. L. Rempe Spring Semester 2006

**Linear Algebra, Geometry and Groups (MATH244)**
**Problem Sheet 7**

Solutions should be handed in on **Monday, March 13th**.

1. Let $q(x, y) = 3x^2 - 6xy + 11y^2$. Find a change of basis which diagonalises $q$ (use the matrix method or complete the squares). Now find an orthogonal change of basis which diagonalises $q$.

2. Consider the quadratic form

$$q(x, y) = x^2 + 5y^2 - 4xy.$$

   Give the symmetric matrix $A$ representing $q$ (with respect to the standard basis). Find a change-of-basis matrix $P$ such that $D = P^T AP$ is diagonal. What are the rank and signature of $q$?

   What type of conic is described by the equation $q(x, y, z) = 5$?

3. Consider the quadratic form

$$q(x, y, z) = x^2 + 6xy + 11y^2 - 2xz - 2yz.$$

   Give the symmetric matrix $A$ representing $q$ (with respect to the standard basis). Find a change-of-basis matrix $P$ such that $D = P^T AP$ is diagonal. What are the rank and signature of $q$?

   What kind of quadric is described by the equation $q(x, y, z) = 7$? Draw a sketch of the surface.

4. Let $q(x, y, z)$ be the quadratic form given by

$$q(x, y, z) = 3x^2 + 3z^2 + 4xy + 8xz + 4yz.$$

   Give the symmetric matrix $A$ representing $Q$ (with respect to the standard basis). Find an *orthogonal* matrix $P$ such that $P^T AP$ is diagonal and hence determine the quadric surface $Q = 4$. Does this quadric have any circular cross-sections?

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