Chaos and Fractals

UNIVERSITY OF LIVERPOOL

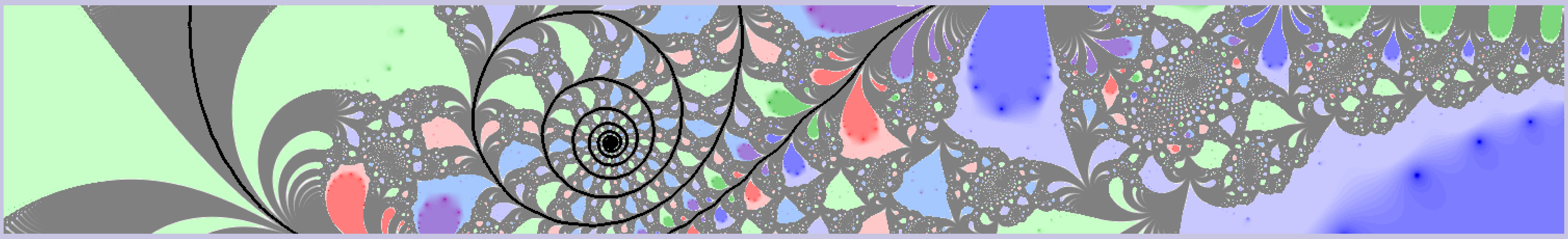
This exhibition accompanies the event "Chaos and fractals - new frontiers" at the 2008 BA Festival of Science and contains computer graphics created by research mathematicians from all around the world. Visitors with or without a mathematical background are invited to enjoy the intrinsic beauty of these images, which arise from a large and exciting area of mathematics.

Dynamical sytems - taming the chaos

Consider the movement of celestial objects in the solar system, or the development of the world's weather. During the second half of the twentieth century, it was discovered that such systems frequently have very irregular, or "chaotic" properties. In particular, any perturbation of the system, no matter how small, can result in vastly different long-term behavior.
Edward Lorenz, the late mathematician and meteorologist, poetically described this effect as follows:

"the flap of a butterfly's wings in Brazil may set off a tornado in Texas months later".

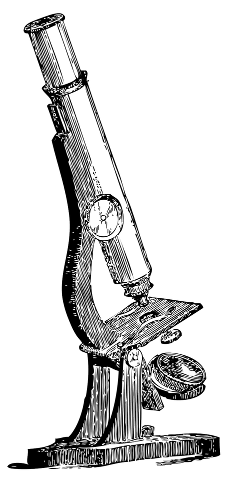
That is a startling prospect for any attempts at accurate predictions: even if we perfectly understood the rules which govern the system, and even if we could measure the current conditions to a precision of several thousand decimal places, that would still not be enough to forecast the long-term behaviour! This explains, for example, why weather reports may be accurate for a day or even two, but predicting the weather weeks or even months in advance is virtually impossible.
The area of mathematics that studies these phenomena is known as dynamical systems theory.



Dynamical Systems at Liverpool

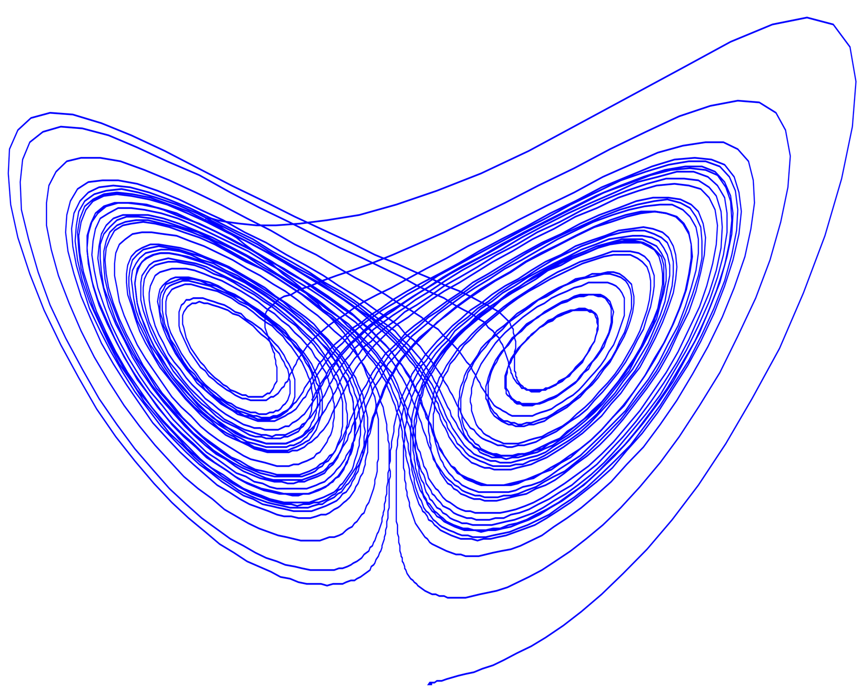
Dynamical systems are one of the main areas of Pure Mathematics represented at the University of Liverpool, and there is a strong and vibrant research group in the field. Aspects of the field are taught in various undergraduate modules, and there are many opportunities for postgraduate studies in the area.

Computer Experiments



Used wisely, a computer is to the mathematician as a telescope is to the astronomer, or a microscope to the biologist. Mathematical systems — even if they are meant to model real-world phenomena — are abstract entities, with precise formal definitions. Frequently, computers can be used to simulate such systems, providing useful observations of their properties.

For example, Edward Lorenz discovered a famous object now called the "Lorenz attractor" while simulating a simplified model of the atmosphere. These experiments helped him and later mathematicians to gain a better understanding of the behaviour of this dynamical system, leading them to phrase conjectures and to eventually establish rigorous mathematical results.

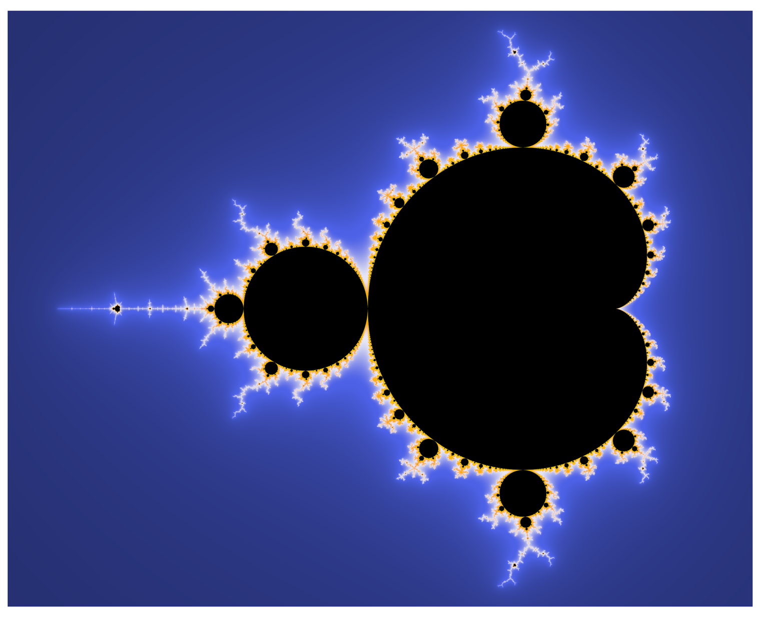


The "Lorenz Attractor"

Often, the outcome of computer experiments is depicted in two- or three-dimensional images, particularly when the object studied is itself two- or three-dimensional, such as the Lorenz attractor. All the images in this exhibition are related to recent or ongoing mathematical research.

The Beauty of Mathematical Images

Apart from aiding with research, computer graphics are also used frequently to communicate mathematical results at conferences and in research articles. As they provide visual representations of abstract concepts, they are also helpful in explaining advanced concepts to undergraduate and postgraduate students.



The "Mandelbrot set"

But in addition, such graphics can be appreciated solely for their aesthetic qualities. The investigation of dynamical systems often produces images of great beauty and complexity. These "fractal" objects have received much attention far outside of mathematics. During the 1980s and 1990s such images were particularly popular, and made their way onto many a poster print - some even appeared as "crop circles"!

Our pictures were, likewise, selected primarily for their aesthetic appeal, and represent a range of current research in dynamical systems. As such, they differ not only from each other but also from some of the more commonly encountered fractal images. All contributors to this exhibition take pride in their images not only for their mathematical accuracy but also their aesthetics, and have invested both their mathematical knowledge and their own sense of beauty into creating the images you see today.

This exhibition was created by Dr. Lasse Rempe, University of Liverpool