# CONTINUED FRACTIONS AND GEOMETRY OF LATTICES

OCTOBER 8, UNIVERSITY OF LIVERPOOL

## Program of the workshop

**11-30** Refreshments.

Session I (Room 604 in penthouse)
**12-00** Imre Bárány (London).
**13-00** Ian Short (Milton Keynes).

**14-00** Coffee break.

Session II (Room G16, MAGIC room)
**14-30** Radhakrishnan Nair (Liverpool).
**15-30** Oleg Karpenkov (Liverpool).
**16-30** Iskander Aliev (Cardiff).

Shortly after the end of the conference (which is 17-00) the participants are kindly invited to take part in the conference dinner.

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## Abstracts

**Imre Bárány**
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### Extremal problems for convex lattice polytopes

In this survey I will present several extremal problems, and some solutions, concerning convex lattice polytopes. A typical example is to determine the minimal volume that a convex lattice polytope can have if it has exactly n vertices. Other examples are the minimal surface area, or the minimal lattice width in the same class of polytopes. These problems are related to a question of V I Arnold from 1980 asking for the number of (equivalence classes of) lattice polytopes of volume V in d-dimensional space, where two convex lattice polytopes are equivalent if one can be carried to the other by a lattice preserving affine transformation.

**Ian Short**
*Open University*
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### Continued fractions and discrete semigroups of Möbius transformations.

We consider the problem of classifying those finite sets of real numbers with the property that every continued fraction with coefficients from one of those sets converges. To tackle this question, we first reformulate it as a problem about the convergence of sequences of Möbius transformations. Using this new framework, we'll see that there are close ties between the original problem and the task of classifying certain discrete semigroups of Möbius transformations. We sketch a solution to the original problem when the set of real numbers has two elements, and indicate the difficulties in dealing with larger sets. Our methods are elementary, using only basic hyperbolic geometry and well-known techniques from the theory of two-generator Fuchsian groups.

**Radhakrishnan Nair**
*University of Liverpool*
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## On the ergodic theory of continued fractions.

We will discuss how ergodic theory can be used to study continued fractions in a a varierty of settings. We will discuss the regular continued fraction, the field of formal power series and the p-adic field in detail. We will then use sub-sequence ergodic theory to deduce new results on their measure theoretic properties and where possible to classify them. This is joint work with Alena Jassova and Poj Lertchoosakul.

**Oleg Karpenkov**
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## Lattice geometry and multidimensional continued fractions

In this talk we will study two approaches to continued fractions via geometric properties of lattices. Both of them have multidimensional versions: Klein polyhedra and Minkowski-Voronoi complexes. We will discuss some recent results and open questions arising in three-dimensional case.

**Iskander Aliev**
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## Polyhedra with prescribed number of lattice points

The well-studied semigroup $\mathrm{Sg}(A) = \{b : b = Ax,\ x \in \mathbb{Z}^n, x \geq 0\}$ can be stratified by the sizes of the polyhedral fibers $IP_A(b) = \{x : Ax = b, x \geq 0, x \in \mathbb{Z}^n\}$. In this talk we discuss a structure theory that characterizes precisely the set $\mathrm{Sg}_{\geq k}(A)$ of all vectors $b \in \mathrm{Sg}(A)$ such that their fiber $IP_A(b)$ contains *at least* $k$ lattice points. We show that, when $n$, $k$ are fixed natural numbers, one can compute in polynomial time an encoding of $\mathrm{Sg}_{\geq k}(A)$ as a generating function, using a short sum of rational functions. Using this tool we prove that for fixed $n, k$ the $k$-Frobenius number can be computed in polynomial time, generalizing a well-known result of R. Kannan.

This is a joint work with Jesus De Loera and Quentin Louveaux.