Partially-ordered homotopy and stratified spaces

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The aim of the project is to develop new algebro-topological techniques for the study of stratified spaces (a wide class of spaces which includes, amongst other things, manifolds, manifolds with boundary and/or with distinguished submanifolds, complex varieties and fibres of analytic maps). Any stratified space has a natural partial order arising from the adjacency diagram of strata, and this partial order is compatible with the underlying topology. The idea of the project is to take account of this extra structure by fusing the notion of a partially-ordered space with homotopy theory.

This is a new area, at present little more than a definition and a few examples exist. This makes the subject easily accessible to a beginning graduate student, and also means that the precise project is quite flexible and can be tailored to the interests and background of the student. Possibilities include investigations of the partially-ordered fundamental groups of

- links in 3-manifolds;
- stratified algebraic curves and their relation to perverse sheaves on the curve;
- the discriminants of singularities and their monodromy action on the Milnor and singular fibres;

or a more theoretical project on

- the fibration sequence for partially-ordered homotopy groups arising from a stratified map and its applications to computations in partially-ordered and standard homotopy theory.

These projects focus on examples which arise in the areas of expertise of other members of the pure mathematics division, namely knot theory, algebraic geometry and singularity theory. Thus there would be ample opportunity for the student to interact fruitfully with other graduate students, with postdoctorates and with members of the department.