Exercise 5 Hint

The easiest way to deal with the condition $f_r(x^*) \neq x^*$ is to notice that you can easily calculate the points x^* with $f_r(x^*) = x^*$ (even by hand) — these are called *fixed points*. So you know how many fixed points there are for each value of r. So you know how many solutions of $f_r^3(x^*) = x^*$ also satisfy $f_r(x^*) = x^*$, and all the rest have $f_r(x^*) \neq x^*$.

In part b), you can also easily determine for which values of r there are fixed points x^* with $\frac{df_r^3}{dx}(x^*)$ between -1 and 1.

To find r_0: suppose you know that two values r_{low} and r_{high} such that $f_{r_{\text{low}}}$ has no period 3 points, and $f_{r_{\text{high}}}$ does have period 3 points (hence r_0 is between r_{low} and r_{high}). You can find a smaller interval that r_0 must live in by considering the average r of r_{low} and r_{high}. If f_r has no period 3 points, replace r_{low} with r, otherwise replace r_{high} with r.

Then the new $[r_{\text{low}}, r_{\text{high}}]$ is half the size of the old one, and still contains r_0. Repeat the process until this interval is small enough that you know r_0 to the required accuracy.