Theorem: 
\( x \) is optimal for the primal LP and \( p \) is optimal for the dual LP \( \text{iff:} \)

\[
\begin{align*}
 p_i (a_i^T x - b_i) &= 0 \quad \forall i \\
(c_j - p^T A_j) x_j &= 0 \quad \forall j
\end{align*}
\]

Proof:
\[
\begin{align*}
 u_i &= p_i (a_i^T x - b_i) \\
v_j &= (c_j - p^T A_j)x_j
\end{align*}
\]

Formal feasibility and dual feasibility imply (proof of Thm 4.3):
\( u_i \geq 0, \quad v_j \geq 0 \)

and:
\[
c^T x - p^T b = \sum_i u_i + \sum_j v_j
\]
all non-negative

Strong duality: \( c^T x = p^T b \) and hence all \( u_i = 0 \) and all \( v_j = 0 \).
This proves the first direction.

Conversely, if all \( u_i = v_j = 0 \), then \( c^T x - p^T b = 0 \), and Cor. 4.2 (weak duality) implies that \( x \) and \( p \) are optimal.
\( \square \)