Let \( x \in \mathbb{R}^n \)

Let \( B(1), \ldots, B(m) \) s.t. \( A_{B(1)}, \ldots, A_{B(m)} \) are lin indep.

\[ x_i = 0 \text{ if } i \notin \{B(1), \ldots, B(m)\} \]

The active constants \( x_i = 0, i \notin \{B(1), \ldots, B(m)\} \) and \( A \cdot x = b \) imply

\[ b = A \cdot x = \sum_{i=1}^{m} A_{B(i)} \cdot x_i = \sum_{i=1}^{m} A_{B(i)} \cdot 0 = 0 \]

Thus, \( x_{B(i)} \) are uniquely determined.

By Theorem 2.2 there are \( n \) lin indep. constants.

By def. \( x \) is a basic solution.

Converse direction:

Let \( x \) be a basic solution.

Let \( x_{B(1)}, \ldots, x_{B(k)} \) the components where \( x \) is nonzero.

Since \( x \) is a basic sol., the system

\[ \sum_{i=1}^{m} A_{c(i)} x_i = b \text{ and } x_i = 0, i \notin \{B(1), \ldots, B(k)\} \]

has a unique solution (Thm 2.2).

Equivalently, the eq. \( \sum_{i=1}^{k} A_{B(i)} x_{B(i)} = b \) has a unique sol.

\[ \Rightarrow \text{columns } A_{B(i)} \text{ are linearly independent} \quad \text{mxk system with lin. indep. columns} \]

Therefore \( k \leq m \)

Since \( A \) has \( m \) lin. indep. rows \( \Rightarrow \) \( A \) has \( m \) lin. indep. cols.

\[ \Rightarrow \text{we can find } \text{m additional columns } B(k+1), \ldots, B(m) \text{ s.t. } A_{B(1)}, \ldots, A_{B(m)} \text{ lin. indep.} \]