COHOMOLOGIES OF GROUPS OF BRAIDS OF SERIES C AND D AND CERTAIN STRATIFICATIONS

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In this note we have computed the cohomologies of groups of braids of series C and D and we have indicated as well the connection of braids of series C with the theory of singularities of smooth functions.

1. Let an irreducible group W, generated by reflections, act in a real n-dimensional Euclidean vector space. Then group W acts also in the complexification C^n of this space. If UW is the union of complexifications of mirrors, then the space $(C^n \setminus UW)/W$ of regular orbits of action of W on C^n is the space $k(\pi, 1)$ (see [1]). The fundamental group of this space is called the group of (generalized) braids connected with group W and is denoted BW (see [2]).

If $W = A_{n-1}$, then BW is the group B(n) of ordinary n-thread braids. Its cohomologies have been computed by Vainshtein [3]. The cohomologies of groups BC_n and BD_n are described by the following two theorems.

THEOREM C. $H^q(BC_n; \mathbf{Z}) = \bigoplus_{i=0}^{\infty} H^{q-i}(B(n-i); \mathbf{Z})$.

THEOREM D. $H^q(BD_n; \mathbf{Z}) = H^q(B(n); \mathbf{Z}) \oplus [\bigoplus_{\mu=0}^{\infty} H^{q-2\mu}(B(n-2\mu); \mathbf{Z})/H^{q-2\mu}(B(n-2\mu-1); \mathbf{Z})] \oplus [\bigoplus_{\lambda=0}^{\infty} H^{q-2\lambda-3}(B(n-2\lambda-3); \mathbf{Z}_2)]$.

Both statements are proved by constructing triangulations of the classifying spaces, analogous to the triangulation in [4]. Multiplicative structures in $H^*(BC_n; Z)$ and $H^*(BD_n; Z)$ are induced by multiplication in the cohomologies of the groups of ordinary braids.

From the stabilization theorem for groups B(n) (see [5]) we obtain

COROLLARY. $H^q(BC_{n+1}; \mathbf{Z}) = H^q(BC_n; \mathbf{Z})$ for $n \geqslant 2q-2$; $H^q(BD_{n+1}; \mathbf{Z}) = H^q(BD_n; \mathbf{Z})$ for $n \geqslant 2q-1$.

Hence for the limit groups (see [6, p. 40])

$$H^q(BC_\infty; \mathbf{Z}) = \bigoplus_{i=0}^{\infty} H^{q-i}(B(\infty); \mathbf{Z});$$

$$H^q(BD_\infty; \mathbf{Z}) = H^q(B(\infty); \mathbf{Z}) \oplus [\bigoplus_{\lambda=0}^{\infty} H^{q-2\lambda-3}(B(\infty); \mathbf{Z}_2)].$$

2. The space $(C^n \setminus UC_n)/C_n$ can be identified with the set of polynomials $p_\lambda(x) = x^n + \lambda_1 x^{n-1} + \dots + \lambda_n$ without multiple and zero roots. The space $C^n/C_n \simeq C^n(\lambda_1, \dots, \lambda_n)$ has a stratification corresponding to the coinciding and vanishing of the roots of $p_\lambda(x)$.

Definition. The set of polynomials in which there are exactly k zero roots and no other multiple roots is called the stratum C_k.

Proposition 1. Stratum C_k $(k \geqslant 1)$ is isomorphic with the space $(C^{n-k} \setminus UC_{n-k}) / C_{n-k}$ and is the space $k(BC_{n-k}, 1)$.

Let f: $(C^m, 0) \to (C, 0)$ be the germ of a function having a normal form D_n, $n \geqslant 4$ (see [7]). Let us consider a miniversal deformation of f (see [7]) and the bifurcation diagram of the zeros of D_n, viz., those points of the base of the deformation that correspond to functions with critical value zero.

Definition. A subset of the base, to which correspond functions with a single critical point at the zero level, where this point is of type D_k, is called a stratum D_k.

Proposition 2. The stratum D_k $(k \geqslant 4)$ of the bifurcation diagram of the zeros of D_n is the space $k(BC_{n-k}, 1)$.

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The assertion is true for one of the irreducible components of strata $(A_1, A_1) = D_2$ and $A_3 = D_3$.

The next proposition is valid for complex caustic D_n (see [7]).

Proposition 3. The stratum D_k $(k \geqslant 4)$ of caustic D_n is the space $k(BC_{n-k}, 1)$.

This proposition is true as well for one of the components of stratum $A_3 = D_3$.

Remark. The Betti numbers of the complement to the complex caustic D_n are: $\beta^0 = \beta^1 = 1$; $\beta^q = 0$, $q > 1$.

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SELF-DUAL YANG–MILLS FIELDS OVER A SPHERE

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1. Let G be one of the classical compact Lie groups: $O(r)$, $U(r)$, $Sp(r)$; P be a principal G-bundle over S^4. In this note we describe all the connectivities on P satisfying the duality equation $*F = F$, where the $*$ is constructed relative to the standard conformal lattice on S^4 and F is a curvature form. In a number of physics papers [1-5, 7, 8] these connectivities are called self-dual Yang–Mills fields or multi-instant solutions of the Yang–Mills equations. For all three series of groups we can establish a one-to-one correspondence between such solutions and certain objects of a linear algebra (the "parameters" of the solution). Following [3] we realize S^4 as a space of planes in C^4, invariant relative to the maps $\sigma: (z_1, z_2, z_3, z_4) \mapsto (-\bar{z}_2, \bar{z}_1, -\bar{z}_4, \bar{z}_3)$. The vector bundle L on S^4, associated with P, is constructed as the direct summand of the trivial bundle $S^4 \times H$ whose rank depends on G and on the Pontryagin number of L. The instant connectivity on L is induced by the trivial connectivity on $S^4 \times H$.

2. Data on a Linear Algebra. In all cases the initial object is the diagram $I \subset H \underset{R}{\otimes} C^4$, where H is a real space with a positive definite quadratic form Q and I is a complex space invariant relative to $\mathrm{id}_H \otimes \sigma$. In addition, in case G = U, on H there must be given an orthogonal operator J with $J^2 = -1$, while in case G = Sp, two orthogonal operators J_1 and J_2 with $J_1^2 = J_2^2 = -1$, $J_1 J_2 = -J_2 J_1$. The subspace I is then assumed invariant relative to $J \otimes \mathrm{id}$ or $J_1 \otimes \mathrm{id}$, and $J_2 \otimes \mathrm{id}$, respectively. The data described must satisfy the following conditions. Let $E \subset C^4 - C$ be a linear space; we set $I_E = \sum_l (\mathrm{id}_H \otimes l) I \subset H \otimes C$, where l ranges the set of linear forms $C^4 \to C^R$, zero on E.

a) For any hyperplane $E \subset C^4$ the space I_E is isotropic relative to $Q \otimes C$.

b) For any σ-invariant plane $D \subset C^4$ we have $\dim_C I_D = 2 \dim_C I$.

3. Bundles. The data described define a bundle $L \subset S^4 \times H$ with fiber $H \cap I_D^{\perp}$ over a point $p \in S^4$, corresponding to plane D. The real rank of L equals $r = \dim_R H - 2 \dim_C I$ and the Pontryagin number equals $n = (1/2)\dim_C I$. The orthogonal (respectively, unitary, quaternion-

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