Local Models for Spatial Analysis
Second Edition

Examples summary: version 1.4

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1. Introduction

This document supports users of the second edition of the book *Local Models for Spatial Analysis* (CRC Press, 2011). It provides guidance on the examples included in the book. The examples included in the first edition of the book (2006) are a subset of those in the second edition and so this workbook also supports users of the first edition. In some cases, data are provided to enable users to test the examples for themselves. In others, links are provided to the relevant software. The data are provided in the same directory as this document.


This document will be revised and expanded over the coming months. If you find this material useful, you would like additional support material, or you identify any problems or errors with the book or this document then please send your comments to Chris Lloyd (c.d.lloyd@liverpool.ac.uk).

The next chapter summarises the worked examples and the following chapters follow the chapter structure of (the second edition of) the book.
2. Worked examples summary

The remainder of this workbook is structured following the chapters in the book *Local Models for Spatial Analysis, Second Edition*. The worked examples are summarised below. Appendix A details software or published sources used to check the worked examples.

**Chapter 3. Grid Data**
1. Fast wavelet transform
2. Slope derivation

**Chapter 4. Spatial Patterning in Single Variables**
3. Geographically weighted summary statistics
4. Global \( I \)
5. Local \( I \)

**Chapter 5. Spatial Relations**
6. Spatial expansion method
7. Geographically weighted regression and \( r \)
8. Geographically weighted ridge regression

**Chapter 6. Spatial Prediction 1: Deterministic Methods, Curve Fitting and Smoothing**
9. Inverse distance weighting
10. Thin plate splines
11. Population surface generation
12. Pycnophylactic method
13. Gregory method

**Chapter 7. Spatial Prediction 2: Geostatistics**
14. Ordinary kriging
15. Kriging with a trend model

**Chapter 8. Point Patterns and Cluster Detection**
16. Quadrat counts
17. \( K \) function and \( L \) function
18. Local \( K \) function and local \( L \) function
19. Besag and Newell cluster analysis

Where relevant, each chapter details files available on the book website which contain data or worked examples.
3. Grid Data

3.1 Fast wavelet transform (FWT)

This example demonstrates the translation of the FWT along the data vector:

17, 26, 29, 43, 59, 80, 71, 59, 61, 60, 58, 58, 61, 75, 69, 63

The full tables follow on the next page (with values given to three decimal places).

The FWT can be computed using the R package wavelets. An example using dwt (wavelets) is attached here: dwtRoutput. Note that the dwt routine starts at the end of the data vector: for the Daubechies 4 wavelet with a vector of length 16 (x1,…,x16) and a periodic boundary, for example, dwt first applies \( c_0 \times x_{15} + c_1 \times x_{16} + c_2 \times x_1 + c_3 \times x_2 \) and then moves two steps up the vector. The approach used in in Local Models for Spatial Analysis (based on numerical recipes (NR) code), starts at the beginning of the vector, and thus the results match if the data vector is entered in reverse (and the coefficients then come out in reverse). If the data vector is entered into dwt the right way round the results do not match at all as the wavelet coefficients are not applied to the same data values in combination. As an example:

NR includes \( c_0 \times x_1 + c_1 \times x_2 + c_2 \times x_3 + c_3 \times x_4 \) as its first operation

dwt includes \( c_0 \times x_4 + c_1 \times x_3 + c_2 \times x_2 + c_3 \times x_1 \) as its second operation

So, the coefficients are applied differently to \( x_1,x_2,x_3,x_4 \) collectively...

3.2 Slope derivation

The worked example is based on the three by three matrix:

<p>| | | |</p>
<table>
<thead>
<tr>
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<td>43</td>
<td>37</td>
<td>35</td>
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</tbody>
</table>

With gradient for the central cell = 4.978 m (78.641°)

As noted on page 68, if the spatial resolution is changed to, for example, 10m (and the denominators become \( 8 \times 10 \)), the gradient is 0.498m (26:464°).
k is position, c is data value and h is the filter coefficient.

1: smooth

<table>
<thead>
<tr>
<th>k</th>
<th>c</th>
<th>h</th>
<th>hc</th>
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SUM 30.895 | 52.848 | 103.695 | 89.553 | 85.146 | 80.497 | 99.513 | 86.471

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SUM -4.536 | -2.674 | 5.331 | -0.363 | -0.837 | -5.701 | 2.588 | 21.387
2: smooth
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1 & 30.895 & 0.483 & 14.921 & 0 & 0 & 0 & 0 & 0.224 & 6.925 \\
2 & 52.848 & 0.837 & 44.208 & 0 & 0 & 0 & 0 & -0.129 & -6.839 \\
3 & 103.695 & 0.224 & 23.243 & 0.483 & 50.081 & 0 & 0 & 0 & 0 \\
4 & 89.553 & -0.129 & -11.589 & 0.837 & 74.913 & 0 & 0 & 0 & 0 \\
5 & 85.146 & 0 & 0 & 0.224 & 19.085 & 0.483 & 41.123 & 0 & 0 \\
6 & 80.497 & 0 & 0 & -0.129 & -10.417 & 0.837 & 67.337 & 0 & 0 \\
7 & 99.513 & 0 & 0 & 0 & 0.224 & 22.305 & 0.483 & 48.061 & 0 \\
8 & 86.471 & 0 & 0 & 0 & 0 & -0.129 & -11.190 & 0.837 & 72.334 \\
\end{array}
\]
\[
\text{SUM} \quad 70.783 & 133.661 & 119.574 & 120.481 \\
\]

2: detail
\[
\begin{array}{cccccccccc}
& k & c & h & hc & h & hc & h & hc & h \\
1 & 30.895 & -0.129 & -3.998 & 0 & 0 & 0 & 0 & 0.837 & 25.844 \\
2 & 52.848 & -0.224 & -11.846 & 0 & 0 & 0 & 0 & -0.483 & -25.524 \\
3 & 103.695 & 0.837 & 86.743 & -0.129 & -13.419 & 0 & 0 & 0 & 0 \\
4 & 89.553 & -0.483 & -43.251 & -0.224 & -20.073 & 0 & 0 & 0 & 0 \\
5 & 85.146 & 0 & 0 & 0.837 & 71.226 & -0.129 & -11.019 & 0 & 0 \\
6 & 80.497 & 0 & 0 & -0.483 & -38.877 & -0.224 & -18.043 & 0 & 0 \\
7 & 99.513 & 0 & 0 & 0 & 0.837 & 83.244 & -0.129 & -12.878 & 0 \\
8 & 86.471 & 0 & 0 & 0 & 0 & -0.483 & -41.762 & -0.224 & -19.382 \\
\end{array}
\]
\[
\text{SUM} \quad 27.648 & -1.143 & 12.420 & -31.939 \\
\]

3: smooth
\[
\begin{array}{cccccccc}
& k & c & h & hc & h & hc \\
1 & 70.783 & 0.483 & 34.186 & 0.224 & 15.866 \\
2 & 133.661 & 0.837 & 111.810 & -0.129 & -17.297 \\
3 & 119.574 & 0.224 & 26.802 & 0.483 & 57.750 \\
4 & 120.481 & -0.129 & -15.591 & 0.837 & 100.784 \\
\end{array}
\]
\[
\text{SUM} \quad 157.206 & 157.103 \\
\]

3: detail
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\begin{array}{cccccccc}
& k & c & h & hc & h & hc \\
1 & 70.783 & -0.129 & -9.160 & 0.837 & 59.211 \\
2 & 133.661 & -0.224 & -29.959 & -0.483 & -64.554 \\
3 & 119.574 & 0.837 & 100.026 & -0.129 & -15.474 \\
4 & 120.481 & -0.483 & -58.188 & -0.224 & -27.005 \\
\end{array}
\]
\[
\text{SUM} \quad 2.719 & -47.822 \\
\]
4. Spatial Patterning in Single Variables

4.1 Geographically weighted summary statistics
Page 75
Datafile: Ch4_GWmean.xls

4.2 Global I
Page 82
Datafile: Ch4_Moran_cell.xls
In the data file, there are two worksheets. The first is for no standardisation and the second is for row standardisation.

4.3 Local I
Pages 92-93
Datafile: Ch4_localmoran.xls
5. Spatial Relations

5.1 Spatial expansion method
Page 111-113
This approach can be implemented in any package which allows for multiple regression.

5.2 Geographically weighted regression and r
Page 125
Datafile: Ch5_GWR.xls (geographical weights for example)

5.3 Geographically weighted ridge regression
Page 132-134
Currently, no public domain software exists for GWRR.
6. Spatial Prediction 1: Deterministic Methods, Curve Fitting and Smoothing

6.1 Inverse distance weighting
Pages 157-158
Datafile: Ch6_IDW.xls

6.2 Thin plate splines
Pages 163-164
Datafile: Ch6_TPScale.xls

An example application of TPS which makes use of the R package fields and compares outputs given manual calculations is given here: TPSzone.txt

6.3 Population surface generation
Pages 178-180 (note that, in the first print run, in both tables the two Y values of 3 should be 2.5)
Datafile: Ch6_PopSurf.xls

The SurfaceBuilder routine by David Martin is available here:
http://www.public.geog.soton.ac.uk/users/martindj/davehome/software.htm

6.4 Pycnophylactic method
Page 183: includes a full worked example

6.5 Gregory method
Page 188: includes a full worked example
7. Spatial Prediction 2: Geostatistics

7.1 Ordinary kriging
Page 210: includes a full worked example

7.2 Kriging with a trend model
Page 225

Given text files ‘KTdata.txt’ and ‘KTdatapred.txt’, the KT system can be solved in S+ (or similarly in R) with:

```r
> KTdd <- matrix( scan("KTdata.txt"), ncol=7, byrow=TRUE)
> KTdd
[1,]  0.00000 167.718 194.20702 218.09773 1 370400 415700
[2,] 167.71847 0.000 215.81657 182.37244 1 370900 412400
[3,] 194.20702 215.817 0.00000 170.47298 1 366100 414700
[4,] 218.09773 182.372 170.47298 0.00000 1 367100 411400
[5,] 1.00000 1.000 1.00000 1.00000 0      0      0
[6,] 370400.00000 370900.000 366100.00000 367100.00000 0      0      0
[7,] 415700.00000 412400.000 414700.00000 411400.00000 0      0      0
[.7]
> KTdp <- matrix( scan("KTdatapred.txt"), ncol=1, byrow=TRUE)
> KTdp
[,1]
[1,] 133.34817
[2,] 151.35985
[3,] 157.43204
[4,] 170.94074
[5,] 1.00000
[6,] 369000.00000
[7,] 414300.00000
> KTwts <- solve(KTdd) %*% KTdp
> KTwts
[,1]
[1,] 4.1671684e-001
[2,] 2.0975415e-001
[3,] 2.7223135e-001
[4,] 1.0129766e-001
[5,] -2.7058517e+003
```
Note that, on page 225 of the (first print run of the) book, there are two typographical errors, but the KT prediction and variance are correct (both cases of -0.000785 should have no minus sign; both cases of 0.000587 should be 0.00587, and so the values in the S+ output are correct).
8. Point Patterns and Cluster Detection

8.1 Quadrat counts
Page 248: includes a full worked example

8.2 K function and L function
Page 254: includes a full worked example
The data for synthetic point patterns A and B are available here: PPA, PPB.

8.3 Local K function and local L function
Pages 262-264: includes a full worked example

Note that the results obtained using PPA 1.0a (Aldstadt et al. 2006) and manual calculations may vary as the former includes an edge correction factor, whereas the example (page 264) does not. Note that there is a typo in the example and -0.243 should read -0.143.

8.4 Besag and Newell cluster analysis
Page 268: includes a full worked example
References


Appendix A: Testing of examples

All of the worked examples have been tested using common software environments, or the results were compared with those reported in published sources. The list provides details of the software used in each case.

**Chapter 3. Grid Data**
1. Fast wavelet transform
2. Slope derivation

**Software/external test:**
Numerical Recipes code (Press et al., 1996)
ArcGIS

**Chapter 4. Spatial Patterning in Single Variables**
3. Geographically weighted summary statistics
4. Global I
5. Local I

**Software/external test:**
R: spgwr (except GW skewness)
GeoDa
GeoDa

**Chapter 5. Spatial Relations**
6. Spatial expansion method
7. Geographically weighted regression and r
8. Geographically weighted ridge regression

**Software/external test:**
Own code/SPSS/LeSage
GWR3.0; SPSS for single example
Wheeler (2007)

**Chapter 6. Spatial Prediction 1: Deterministic Methods, Curve Fitting and Smoothing**
9. Inverse distance weighting
10. Thin plate splines
11. Population surface generation
12. Pycnophylactic method
13. Gregory method

**Software/external test:**
ArcGIS
R: fields
Own code
Tobler (1979)
Own code (example in Gregory and Ell, 2005)

**Chapter 7. Spatial Prediction 2: Geostatistics**
14. Ordinary kriging
15. Kriging with a trend model

**Software/external test:**
GSLIB (1992 version)
GSLIB (1992 version)

**Chapter 8. Point Patterns and Cluster Detection**
16. Quadrat counts
17. K function and L function
18. Local K function and local L function
19. Besag and Newell cluster analysis

**Software/external test:**
Boots and Getis (1988)
Own code
PPA 1.0a (Aldstadt et al. 2006)
R: DCluster

**Number**
1,2,3,4,5,7,8,9,10,11,12,14,15,16,17
13,18,19
6

**Date checked**
28th June 2010
29th June 2010
9th July 2010

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