Physica D 49 (1991) 107–113
North-Holland

Vortex initiation in a heterogeneous excitable medium

A.V. Panfilov and B.N. Vasiev
Institute of Biological Physics, 142292, Pushchino, Moscow Region, USSR

We studied numerically the process of vortex initiation in the heterogeneous active medium which is described by a FitzHugh–Nagumo-type model. Vortex initiation results from interaction of two external stimuli with a stepwise inhomogeneity in refractoriness. The influence of distance between the place of stimulation and heterogeneity and geometrical sizes of the heterogeneity on the process of vortex initiation is examined. The drift and interaction of vortices is also studied.

1. Introduction

The existence of vortices is a general property of excitable media of different nature. Their appearance in physical, chemical and biological media leads to a special kind of chaos in these media – autowave chaos [1]. The initiation of vortices in cardiac tissue leads to a dangerous cardiac arrhythmias, in particular, to paroxysmal tachicardia and to heart fibrillation [2].

One of the most important mechanisms of the vortex initiation was proposed by Krinsky in 1968 [3], who studied an axiomatic model of a heterogeneous excitable medium (fig. 1). In this axiomatic model each point of the medium can be in the following three states: state of rest, excitation state and refractory state. The point at rest becomes excited if any of neighboring points is in the excited state. In this model, in contrast to the classical Wiener–Rosenbluth model [4], the excited state lasts for some finite period of time, usually referred as τ. In ref. [3] the following process of vortex initiation was proposed. Assume that in an excitable medium with a refractoriness R_l a region D with a prolonged period of refractoriness R_h exists. If in this medium two waves propagate with an interval T_{st} (which has a value between R_l and R_h), then a second wave cannot penetrate into the region D, because its refractoriness R_h is higher than T_{st}. As a result, a wave break occurs. This break moves along the heterogeneity boundary, and when the refractoriness period in the region D ends, the break propagates into this region and a spiral wave is formed.

This mechanism has been confirmed in experiments on cardiac tissue [5], and it is perhaps one of the main mechanisms of the initiation of the cardiac arrhythmias.

In this paper we consider this process of vortex initiation in a reaction–diffusion model of an excitable medium.

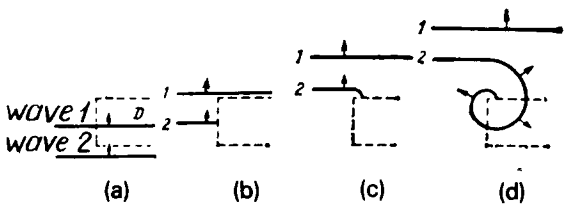

Fig. 1. Vortex initiation in an axiomatic model of an excitable medium.

2. Model, method of computation, and results

To represent the excitable medium the FitzHugh–Nagumo model was used:

$$\frac{\partial E}{\partial t} = \Delta E - f(E) - g, \quad \frac{\partial g}{\partial t} = \varepsilon(E)(E-g), \tag{1}$$

where Δ is the two-dimensional Laplacian, $f(E)$ is the nonlinear N-shaped function, $\varepsilon(E)$ is the parameter determining the temporal behavior of the slow variable g, i.e. the duration of the excited state and the duration of the refractory tail of the wave.

For calculations the following shapes of functions $f(E)$ and $\varepsilon(E)$ were used:

$$\begin{aligned} f(E) &= C_1 E, && E < E_1, \\ &= -C_2(E-a), && E_1 < E < E_2, \\ &= C_3(E-1), && E > E_2, \\ \varepsilon(E) &= \varepsilon_1, && E \le 0, \\ &= \varepsilon_2, && 0 < E < 1, \\ &= \varepsilon_3, && E \ge 1, \end{aligned} \tag{2}$$

where the parameters determining the shape of the function $f(E)$ were as in a previous paper [6]: $E_1 = 0.018$, $E_2 = 0.94$, $C_1 = 4$, $C_2 = 1$, $C_3 = 15$, $a = 0.09$. These parameters specify the fast processes such as initiation and propagation of the wave front. The main parameter for the purpose of our paper is refractoriness, which is determined by function $\varepsilon(E)$. In $\varepsilon(E)$ (see eq. (2)) the parameter ε_1 specifies the duration of the refractory tail and ε_3 the duration of the excited state. To isolate the effects of refractoriness, the value of ε_1 was chosen smaller than ε_3; in particular, the values $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.05$ and $\varepsilon_3 = 2$ were used.

The inhomogeneity was preset by the parameter ε_1. In our calculations the value of ε_1 in region D changes from 0.2 to 0.03.

Neumann boundary conditions were used. Calculations were performed using the explicit Euler method with standard space step $h_x = 1.2$ and time step $h_t = 0.12$ [6].

2.1. Onset of the wave break

In the axiomatic model there are the following conditions for the break formation [3]:

$$R_l < T_{st} < R_h - \tau, \tag{3}$$

where T_{st} is the time interval between stimulations, R_l is the refractoriness period of the medium, R_h is the refractoriness in the D region, and τ is the duration of the excitable state. The width of the interval (3) in which the break could be formed is usally referred to as the width of the vulnerable phase.

In reaction–diffusion models where the velocity of the first wave is greater than that of the second one the condition of the break formation becomes

$$R_l < T_{st} < R_h - \tau - (t_2 - t_1), \tag{4}$$

where

$$t_1 = \int_0^L \frac{dx}{V_1} = \frac{L}{V_1}, \quad t_2 = \int_0^L \frac{dx}{V_2(l)} \tag{5}$$

are the times during which the first (t_1) or the second (t_2) wave propagates a distance L from the place of stimulation to the region D (fig. 2a), and x is the current distance. The $V_2(l)$ is the dispersion relation (the dependence of the velocity of the second wave on its distance l to the first wave).

The delay of the second wave from the first leads to a decrease in the width of the vulnerable phase, ΔT. In fact, the velocity of the second wave is smaller than that of the first one and the distance between them increases with time. This also formally follows from (4), (5), as the difference between the two integrals has a constant sign and hence $t_2 - t_1$ increases with increasing L. At a sufficiently large L the width of the vulnerable phase, ΔT, can be equal to zero even if heterogeneity in refractoriness ΔR exists.

Fig. 3 shows the dependence of the width of the vulnerable phase, ΔT, on the distance L which was obtained numerically. It is seen that the value of ΔT decreases to zero with an increase of the value of L.

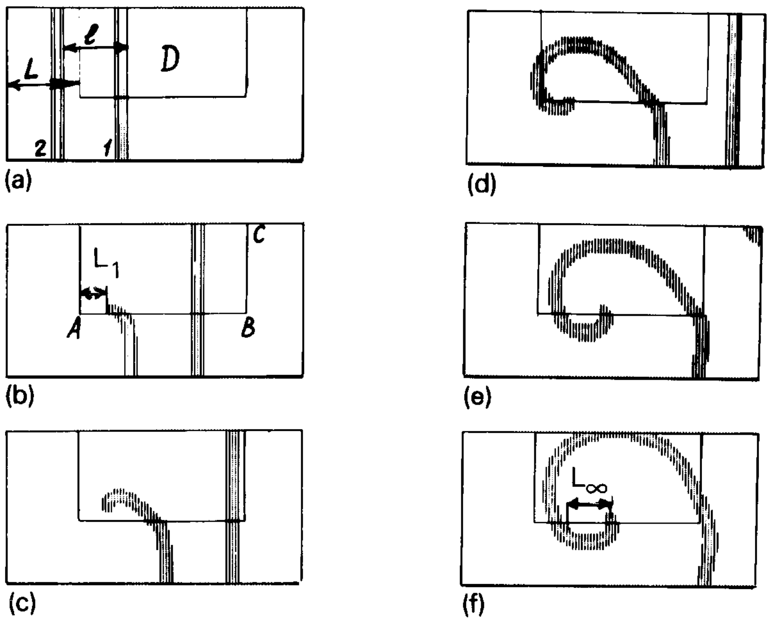

Fig. 2. Vortex initiation in a reaction–diffusion model. Numerical simulations. Model (1). In the dark regions $E > 0.8$, ε_1 in the medium is equal to 0.2; in the region D, $\varepsilon_1 = 1/17.5$. (At $\varepsilon_1 = 0.2$ the wavelength is $\lambda = 24$ and the period of the vortex is $T = 17$; at $\varepsilon_1 = 1/17.5$, $\lambda = 32$, $T = 43$.) The size of the heterogeneity is 36×66. In the successive figures the time is as follows: (a) $T = 43$, (b) $T = 71$, (c) $T = 86$, (d) $T = 100$, (e) $T = 215$, (f) $T = 532$.

2.2. *Generation of a vortex*

For the vortex to arise, the break of a front of the second wave has to penetrate into the region D. This penetration in the τ-model can happen only on the boundary BC (see fig. 2b) of region D, as the velocities of the first and the second waves are equal. In reaction–diffusion models, this can happen also on the boundary AB as the velocity of the second wave is smaller than that of the first one.

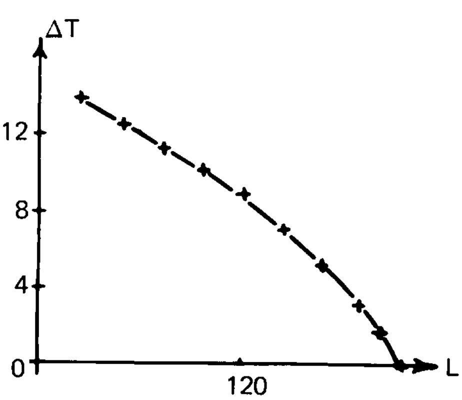

Fig. 3. The dependence of the vulnerable phase width, ΔT, on the distance L between the heterogeneity D and the region of stimulation (see fig. 2). $R_l = 12.6$, $R_h = 25.8$.

Fig. 2 shows this process. It is seen that the distance between the first and the second wave successively increases, and when it becomes more than the refractoriness in D, the second wave penetrates in region D and forms the vortex.

The distance L_l on which the second wave propagates along the boundary before penetrating into the region D is determined at least by two factors: the lag of the second wave from the first one, and the bending of the front of the second wave. One can see this bend in fig. 2b. The bend of the second wave is associated with the fact that the wave near the break has a smaller velocity than the plane wave due to the leakage current at the break.

If the length of the boundary AB is less than the distance L_l, the excitation wave penetrates

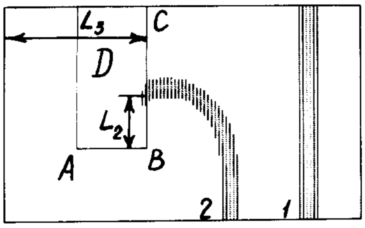

Fig. 4. Vortex generation in a dynamic model (1) in the case of $L_l > AB$. $R_l = 12.6$; $R_h = 43.8$.

into the region D on the boundary BC (fig. 4). This case is similar to that in the axiomatic model (fig. 1) where the breaking wave propagates a distance $L_2 = (R_h - T_{st})V$ along the boundary BC and penetrates into region D and forms a spiral.

Let us estimate the path L_2 in the case of dynamic models. If the distance between the place of stimulation and the region D is equal to L_3 (fig. 4), then similar to (4), (5) we have:

$$L_2 = \left(R_h - T_{st} + \frac{L_3}{V_1} - \int_0^{L_3} \frac{dx}{V_2(l)}\right) V_1. \tag{6}$$

In (6) we take into account only one factor: the lag of the second wave from the first one on the path L_3; we do not consider the bend of the second wave and its dispersion on the boundary BC. If the length of the border BC is less than L_2 the vortex cannot arise and the breaking wave runs into the boundary of the medium and disappears.

2.3. *Drift of the vortex*

Numerical experiments have shown that the vortex arising on the stepwise inhomogeneity drifts (fig. 2). The velocity of the drift has two components: along the boundary of heterogeneity and transverse to it. Due to the transversal component the vortex shifts into the region D, its drift velocity decreases, and the vortex stops at some distance L_∞ from the point of origin.

The value of the distance L_∞ depends on the extent of inhomogeneity of the medium $\Delta R/R_l$. Fig. 5 shows the dependence of L_∞ on $\Delta R/R_l$. It is seen that the distance L_∞ increases exponentially with an increase of the degree of inhomogeneity of the medium.

Calculations have shown that the direction of longitudinal drift of the vortex depends on the direction of its rotation and the direction of the transversal drift, and it is given by the vector:

$$\boldsymbol{U} = [\boldsymbol{\omega} \times \boldsymbol{T}], \tag{7}$$

where $\boldsymbol{\omega}$ is the vector of angular velocity of the vortex and $\boldsymbol{T}$ is the vector in the direction of the transversal drift.

Interaction of two drifting vortices. In the case when the region D is within the medium (fig. 6), two vortices arise on the heterogeneity. These vortices rotate in opposite directions, and according to formula (7) will drift towards each other. Numerical experiments have shown that due to this drift the annihilation of the vortices can occur (fig. 6), i.e., vortices arising on heterogeneities of excitable media could have a finite lifetime.

Fig. 7 shows the dependence of the vortices' lifetime, T_l, on the initial distance between them. It is seen that the lifetime increases with an increase of the distance. When the initial dis-

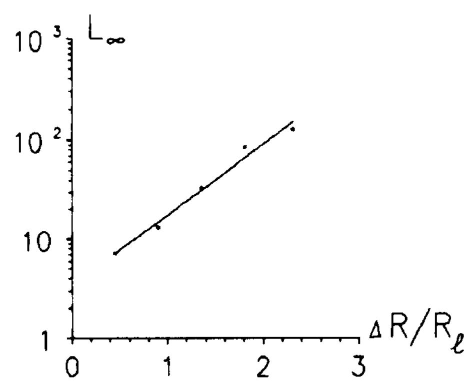

Fig. 5. The dependence of the distance of the vortex shift, L_∞, on the extent of inhomogeneity of the medium, $\Delta R/R_l$. $R_l = 12.6$, $R_h = 12.6$–43.8. L_∞ is given on a logarithmic scale.

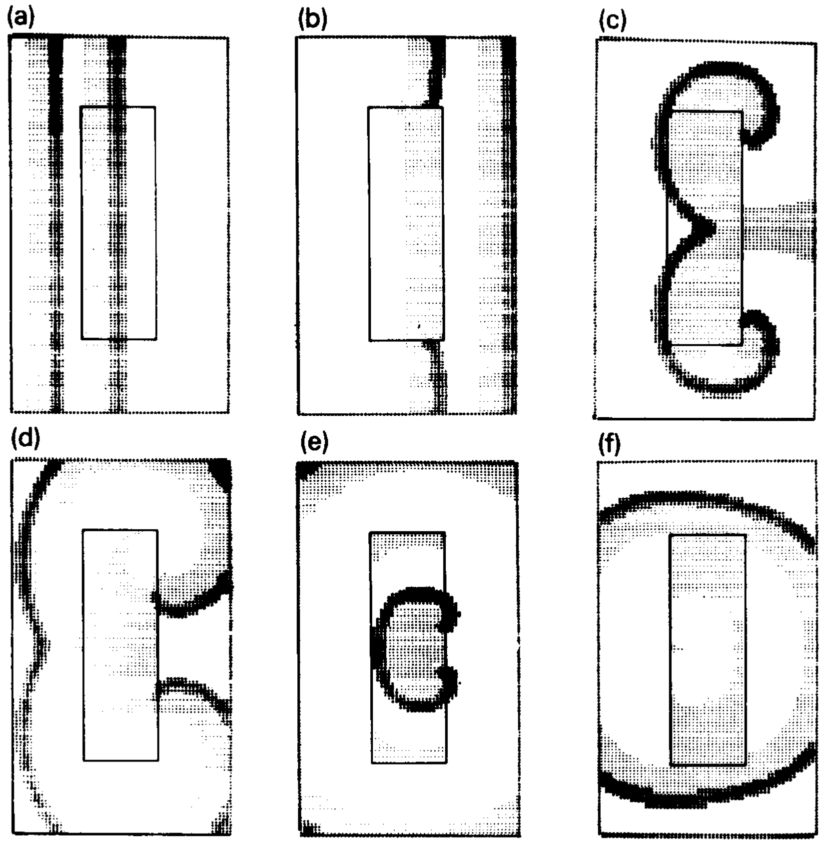

Fig. 6. The initiation of two vortices and their annihilation. $R_\iota = 12.6$, $R_h = 33.6$. (a) $T = 33$, (b) $T = 67$, (c) $T = 117$, (d) $T = 134$, (e) $T = 418$, (f) $T = 451$.

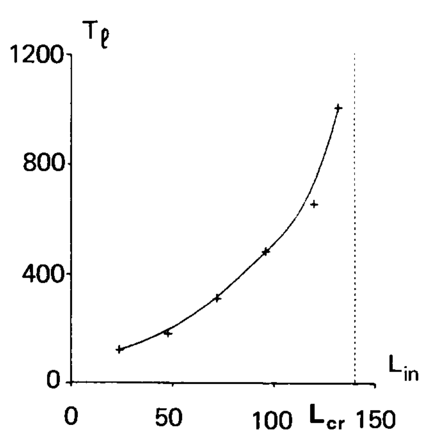

Fig. 7. The dependence of the lifetime of the vortices, T_ι, on the initial distance between them, L_{in}. $R_\iota = 12.6$, $R_h = 30.0$.

tance is more than L_{cr} the lifetime becomes equal to infinity, i.e., due to the initial conditions two different regimes of vortex interaction could be realized: the regime of annihilation or the regime of the infinite lifetime.

In the absence of annihilation the vortices stabilize at some distance L_f between them. It is obvious that in the case of a large initial distance ($L_{in} \gg L_{cr}$) the following formula for the final distance L_f is valid:

$$L_f = L_{in} - 2L_\infty. \tag{8}$$

The dependence of L_f on L_{in} in the whole range is shown in fig. 8. It is seen that at large L_{in} the dependence is linear, in accordance with for-

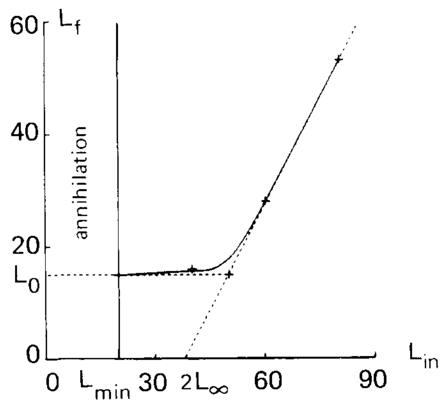

Fig. 8. The dependence of the final distance between vortices, L_f, on the initial distance between them, L_{in}. $R_l = 12.6$, $R_h = 22.8$.

mula (8). At small L_{in} the dependence departs from linearity and tends to some value L_0. If the initial distance between vortices is less than L_{min}, the vortices annihilate. Further investigations have shown that L_0 is the minimal possible distance between two vortices in homogeneous excitable media in which they can exist. The existence of this distance has been found in ref. [7], where it has been shown that two vortices rotating in opposite directions repel each other and diverge to some distance L_0.

3. Discussion

Some experimental results on vortex behavior in heterogeneous cardiac tissue [8] and in a temperature gradient in the Belousov–Zhabotinsky reaction [9] have been obtained recently. Let us compare them with the results of the present paper. The experimental conditions in ref. [8] are analogous to those in the present paper: a stepwise heterogeneity in refractory period, and a drift of the vortex along the boundary of heterogeneity does exist. The direction of this drift coincides with the direction of the longitudinal drift of the vortex shown in fig. 2, i.e., with that given by formula (7). Unfortunately, the transversal component of the drift has not been observed in ref. [8]. This can be due to the small size of the fragment of cardiac tissue studied in that paper; the vortex ran into the wall and disappeared after two rotations.

In ref. [9] both components of the vortex drift were observed. The direction of the longitudinal component also coincides with that given by formula (7), but the direction of the transversal component is the opposite: in ref. [9] the vortex drifts so that its period decreases, whereas in the present paper the drift is directed to the increase of the vortex period. This difference is probably associated with the fact that the temperature gradient in the BZ reaction produces not only heterogeneity in refractoriness but also in the velocity of the wave propagation, while in the present paper heterogeneity in velocity is absent.

Finally, we point out one of the results which could be interesting for experiments in cardiac tissue. As shown in formulae (4), (5), the conditions of the break formation depend on the distance from the place of stimulation to the region of heterogeneity. This effect is due to the dependence of the wave velocity on the distance between the waves. This dependence is well known for cardiac tissue [10]. Therefore, we can assume that for cardiac tissue the width of vulnerable phase depends on the distance from the place of stimulation to the region of heterogeneity. This also means that the extent of heterogeneity of excitable medium obtained in experimental measurements will depend on the distance from the stimulating electrode to the boundary of heterogeneity.

References

[1] V.I. Krinsky, Autowaves: results, problems, outlooks, in: Self-Organization. Autowaves and Structures Far from Equilibrium, ed. V.I. Krinsky (Springer, Berlin, 1984) pp. 9–19.

[2] V.I. Krinsky, Pharmac. Theor. B3 (1978) 539–544.

[3] V.I. Krinsky, Problem. Kibern. 20 (1968) 59–80.
[4] N. Wiener and A. Rosenbluth, Arch. Inst. Cardiol. Mexico 16 (1946) 205–265.
[5] M.A. Allesie, F.I.M. Bonke and F.J.G. Schopman, Circ. Res. 39 (1976) 168–177.
[6] A.V. Panfilov and A.N. Rudenko, Physica D 28 (1987) 215–218.
[7] E.A. Ermakova, A.M. Pertsov and E.E. Shnol, Physica D 40 (1989) 185–195.
[8] V.G. Fast and A.M. Pertsov, Circ. Res. (1990), in press.
[9] K.I. Agladze, A.V. Panfilov and B.N. Vasiev, Physica D, submitted for publication.
[10] J.L.R.M. Smeets, M.A. Allessie, W.J.E.P. Lammers, F.I.M. Bonke and J. Hollen, Circ. Res. 58 (1986) 96–108.