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The Drift of a Vortex in an Inhomogeneous System of two Coupled Fibers

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Abstract—We studied the behavior of a vortex in an excitable medium having a stepped inhomogeneity, which is represented by a system of two coupled fibers. Numerical experiments were performed and analytical expressions were obtained for the determination of vortex drift velocity as a function of the parameters in the FitzHugh–Nagumo model.

INTRODUCTION

This paper is concerned with the behavior of a vortex in an inhomogeneous excitable medium that consists of two nonidentical coupled fibers. The existence of rotating vortices is common to a great variety of active media. Vortices in physical, chemical and biological media are the main cause of instabilities in normal wave regimes and of chaos in the initially ordered media. The most dangerous disturbances of cardiac rhythm—paroxysmal tachycardia and fibrillation—are associated with the appearance of vortices [1].

The simplest possible medium where vortices can occur is a system of two coupled fibers [2, 3]. In particular, such a system can be formed by the Purkinje fibers in the heart or by trabeculae in the myocardial tissue [4]. The vortex in this system is an impulse rotating along a closed trajectory. In a previous work [5], the properties of vortices were studied theoretically and numerically for the case of a homogeneous medium. However, the vortex behavior in inhomogeneous media is important because it is at inhomogeneities that vortices originate and vanish [6].

In a system of two coupled fibers, two basic types of inhomogeneity attract the greatest interest, longitudinal, along the length of identical fibers, and transversal, when both fibers are homogeneous but differ in their properties, for instance, in refractoriness. Systems having a transversal inhomogeneity are analogous to two-dimensional media with a stepped inhomogeneity.

The behavior of a vortex in a system with a longitudinal inhomogeneity was studied in [3]. Such a vortex was shown to drift in the direction of its increased period and the drift mechanism was investigated. Systems with a transverse inhomogeneity have been much less well studied. It has only been shown [7] that the drift of a vortex in such systems is, in principle, possible, but no detailed investigation was carried out.

The present work is aimed at studying systematically the behavior of a vortex in a system of two coupled fibers with a transversal inhomogeneity. We consider two types of inhomogeneity, with respect to medium refractoriness and with respect to excitation propagation velocity, for different impulse duration and coupling coefficients. We present an axiomatic approach for the drift of a vortex (Section 1), the results of numerical investigations (Section 2) and analytical estimates for the vortex drift velocity in the FitzHugh–Nagumo model (Section 3).

1. AXIOMATIC APPROACH TO A PROBLEM OF VORTEX DRIFT IN A SYSTEM OF TWO COUPLED FIBERS WITH A TRANSVERSAL INHOMOGENEITY

Consider what happens to a vortex in a system of two coupled fibers, proceeding from its simplest properties. As follows from [2], such a system is characterized by the following basic parameters: velocity of a single wave, V_{sing}, velocity of a collective wave, V_{coll}, and medium refractoriness, R. Consider first the case of a homogeneous medium (Fig. 1).

Let a wave propagate only along fiber 1. At a certain moment we connect the two fibers and the wave of excitation passes to the second fiber (point A), where two oppositely propagating waves (1 and 2) are thus formed (Fig. 1c). Wave 1, together with the wave of the first fiber, forms a collective wave and wave 2 remains single. After some time, the single wave will be able to excite the first fiber (point B in Fig. 1d). This will occur when the first fiber just beneath the single wave recovers from refractoriness. After point B is excited, two waves will originate from it (Fig. 1e), one of them being single again. After time T equal to the refractory period of the medium at point B, the wave will come back to point A and will be able to excite two waves on the second fiber.

Thus, in a homogeneous system of two coupled fibers a vortex is a single wave circulating between points A and B (Fig. 1) at velocity V_{sing} and period T equal to the medium refractory period R. If the fibers differ in refractoriness, the steady state circulation of the wave becomes impossible [2]. Let us describe this process at greater length.

Let the refractoriness of fiber 1 (Fig. 2) be R_1 and of fiber 2, R_2, $R_1 < R_2$. The single wave then makes a cycle from point A to point B and back for time R_1 and has to run further along the first fiber, since $R_2 > R_1$. By the time it has returned to point A, the second fiber has not yet recovered (Fig. 2). For the single wave to be able to pass again to the second fiber, the latter must recover from refractoriness. This occurs only if the interval between single wave 1 and collective wave 2 (see Fig. 2) reaches the refractoriness value of

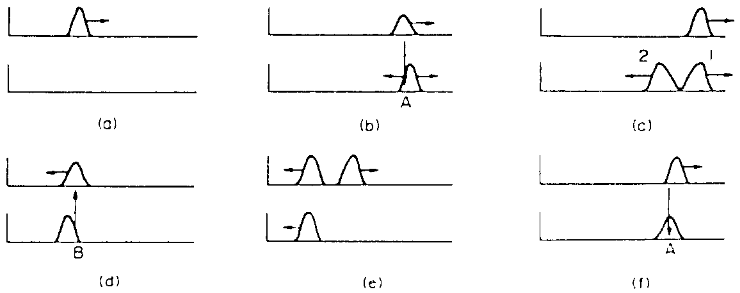

Fig. 1. A vortex in a homogeneous system of two coupled fibers.

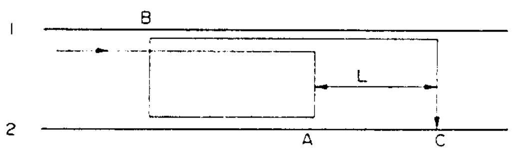

Fig. 2. Drift of a vortex in a system of two coupled fibers with unequal refractoriness.

the second fiber, R_2, i.e. only if the single wave lags behind the collective one, $V_{sing} < V_{coll}$. In the case of $V_{sing} = V_{coll}$, for example in axiomatic models of excitable media [6], wave 1 can never pass to the second fiber and the vortex vanishes after emitting just one wave. As shown theoretically and numerically [5], in realistic models of excitable media $V_{sing} < V_{coll}$ and, therefore, wave 1 covers a distance L along the first fiber and then passes to the second one. The process described recurs in each turn-over of the single wave and causes the vortex to drift.

The drift of a vortex in a system of two coupled fibers differing in refractoriness was considered in [2] and an equation for the drift velocity was obtained. Here we outline the derivation of this equation.

The distance L travelled by a single wave during one period is easily found to be:

$$L = \Delta R^* V_{coll}{}^* V_{sing}/\Delta V, \tag{1.1}$$

where $\Delta R = R_2 - R_1$, $\Delta V = V_{coll} - V_{sing}$.

The time interval between successive passages of the single wave to the second fiber (at point A and C) is

$$T = R_1 + \Delta R^* V_{coll}/\Delta V. \tag{1.2}$$

Given L and T, the drift velocity, V_{dr}, in the system considered can easily be obtained

$$V_{dr} = L/T = V_{sing}/(1 + \Delta V^* R/(\Delta R^* V_{coll})). \tag{1.3}$$

It follows from the above consideration that the direction of the drift is determined by the following vector relation

$$\mathbf{V}_{dr} = [\omega^* \mathbf{grad}\ R], \tag{1.4}$$

where ω is the vector of the angular velocity of vortex rotation and **grad** R is the vector orthogonal to the fiber and directed toward the greater refractoriness.

If the system under examination is, in addition, inhomogeneous with respect to excitation propagation velocity, i.e. the velocity of a single wave in the first fiber, V_{sing1}, differs from that of the second fiber, V_{sing2}, simple analysis shows that equation (1.3) remains valid but one should substitute into it V_{sing1} for the case of Fig. 2 or V_{sing} of the fiber whose refractoriness is lower in the general case:

$$V_{dr} = V_{sing1}/(1 + \Delta V^* R/(\Delta R^* V_{coll})). \tag{1.5}$$

From (1.5) it follows that:

1. If the fibers differ only in wave velocities and not in refractoriness ($\Delta V \neq 0$, $\Delta R = 0$), then $V_{dr} = 0$, i.e. the vortex rotates steadily and does not drift.

2. Velocity V_{dr} increases with increasing degree of inhomogeneity, $\Delta R/R$, and at large $\Delta R/R$ it reaches a saturated level:

$$V_{max} = V_{sing}. \tag{1.6}$$

3. The drift velocity increases with decrease in the difference between the value V_{sing} and V_{coll} and when $\Delta V/V_{coll} \to 0$ the drift velocity $V_{dr} \to V_{sing}$.

Equation (1.5) describes the dependence of V_{dr} on other important parameters of an excitable medium also, such as the coefficient of coupling between the fibers, P, and the duration of the excited state, τ.

4. Velocity V_{dr} must fall with increasing P. Indeed, the increased coupling between the fibers, according to [2, 5], leads to a decrease of V_{sing} and therefore, by equation (1.5), to a decrease in V_{dr}.

5. Velocity V_{dr} is independent of the duration of the excited state, τ.

6. A drifting vortex, just as a stationary one, 'emits' collective waves. However, in the case of a drifting vortex, the frequency of collective waves in different directions is not the same. In the direction of drift of a vortex, collective waves follow with a period equal to the larger refractory period:

$$T_f = R_2 \tag{1.7}$$

In the opposite direction, the period of collective waves is longer and depends on V_{dr}, according to Doppler's effect [2]:

$$T_b = R_2^*(1 + V_{dr}/V_{coll})/(1 - V_{dr}/V_{coll}) \tag{1.8}$$

To verify these conclusions, numerical experiments were carried out, which are described below.

2. RESULTS OF NUMERICAL EXPERIMENTS

Here we present the results of numerical experiments on a FitzHugh–Nagumo model describing a system of two coupled fibers [3]. We considered a simple ohmic coupling between the fibers. The active medium under study is a system of equations:

$$\begin{aligned}
\frac{\partial E_1}{\partial t} &= \frac{\partial^2 E_1}{\partial x^2} - f(E_1) + P^*(E_2 - E_1) - g_1 \\
\frac{\partial g_1}{\partial t} &= \varepsilon(E_1)^*(E_1 - g_1) \\
\frac{\partial E_2}{\partial t} &= \frac{\partial^2 E_2}{\partial x^2} - f(E_2) + P^*(E_1 - E_2) - g_2 \\
\frac{\partial g_2}{\partial t} &= \varepsilon(E_2)^*(E_2 - g_2)
\end{aligned} \tag{2.1}$$

where P is the parameter determining the coupling between the fibers, E and g are the variables characterizing the state of each point of a fiber, $f(E)$ is the nonlinear N-shaped function, $\varepsilon(E)$ is the parameter determining the temporal behavior of the slow variable g, i.e. the duration of the excited state and the duration of the refractory tail of the wave.

The calculation was performed for the piecewise linear function $f(E)$: $f(E) = -C^*(E - a)$ if $0 < E < 1$, $E = 0$, if $E \leq 0$, and $E = 1$ if $E \geq 1$, (where $C = 1$, $a = 0.085$). This shape of function arises as a limit transition for function

$$f(E) = \begin{cases} S^*E & \text{when } E \leq 0 \\ -C^*(E - a) & \text{when } 0 < E < 1 \\ S^*(E - 1) & \text{when } E \geq 1 \end{cases} \tag{2.2}$$

where S tends to infinity. As we shall see later (Section 3.) this shape is very convenient for analytical studying.

The parameter $\varepsilon(E)$ was defined as follows:

$$\varepsilon(E) = \begin{cases} \varepsilon_1 & \text{when } E \leq 0 \\ \varepsilon_2 & \text{when } 0 < E < 1 \\ \varepsilon_3 & \text{when } E \geq 1 \end{cases} \tag{2.3}$$

The parameter ε_1 specifies the duration of the refractory tail, ε_3 the duration of the excited state and $\varepsilon_2 \ll 1$.

A more detailed description of the model and of the experimental procedure is given in a previous work [5]. The model was made inhomogeneous either in the quantity ε_1 characterizing the medium refractoriness or in the diffusion coefficient D determining the velocity of wave propagation. Numerical experiments were performed using Neuman boundary conditions $\partial E/\partial n = 0$. Numerical integrations in a rectangular coordinate system were by the explicit Euler method with a space step of $\Delta x = 0.5$ and time step of $\Delta t = 0.04$.

The vortex was obtained as follows: an excitation wave was initiated to propagate along one fiber (Fig. 1a). After some period of time the coupling between the fibers was switched on. As a result, the excitation wave invaded to the second fiber (point A) (Fig. 1b) and two excitation waves (1 and 2) moving in opposite directions along fiber 2 occur (Fig. 1c). The wave on the first fiber and wave 1 (Fig. 1c) is called a collective wave; wave 2 is called a single reflected wave. With the further passage of time, the single wave will be able to reinvade the first fiber (point B in Fig. 1d). This will occur when the first fiber directly opposite the single wave has recovered and is able to be re-excited. After excitation of point B, two waves on the first fiber appear (Fig. 1e). One of these waves, again, will be single. In time this wave, again, will be able to generate an excitation wave on the second fiber (Fig. 1f).

Thus, in a system of two coupled fibers, the vortex is a single wave rotating along a closed trajectory. The duration of one cycle is the period of the vortex, and region of the vortex circulation (the distance between the point A and B) is the vortex size.

Numerical experiments have shown the following:

1. A vortex in a system of two coupled fibers with different refractoriness displays drift. The drift direction depends on the direction of rotation and is determined by equation (1.4).

2. The drift velocity V_{dr} grows with increasing inhomogeneity of the medium. In Fig. 3 the velocity V_{dr} is plotted against the quantity $\Delta R/R$ characterizing the degree of inhomogeneity in refractoriness. It is seen that V_{dr} grows with $\Delta R/R$ and reaches a saturation at $\Delta R/R \approx 1$. The shape of the plot fits equation (1.3).

3. Velocity V_{dr} is independent of the duration of the excited state. Figure 3(a) presents a family of curves for various durations of the excited states, τ. It is seen that V_{dr} remains

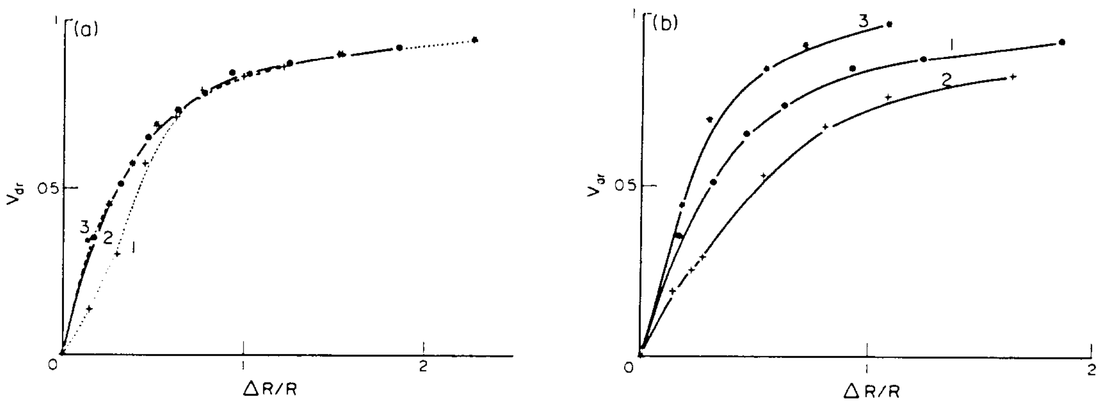

Fig. 3. Vortex drift velocity as a function of the degree of inhomogeneity on refractoriness $\Delta R/R_1$. (a) For various durations of the excited state: 1. $\tau = 0.7$ ($R_1 = 35.2$); 2. $\tau = 6.9$ ($R_1 = 41.4$); 3. $\tau = 13.8$ ($R_1 = 48.3$); $P = 0.15$. (b) For various coefficients of coupling between fibers: 1. $P = 0.15$; 2. $P = 0.2$; 3. $P = 0.1$ ($R_1 = 41.4$ for $P = 0.15$, $\tau = 6.9$).

practically unaltered when of the duration of the excited state increases by more than a factor of 10. This is also in accord with the results of qualitative theory.

4. Velocity V_{dr} falls with an increase in the coupling coefficient. In Fig. 3(b), there is shown a family of the curves characterizing the dependence of V_{dr} on $\Delta R/R$ for different values of the coupling coefficient P. It is seen that a 1.5- or 2-fold increase in P causes the velocity to drop by 20–40%. The maximum velocity also decreases at a high degree of inhomogeneity, which agrees qualitatively with the results of Section 1.

5. As distinct from theory, in numerical experiments the drift of a vortex in a medium having a velocity gradient was obtained even in the absence of a inhomogeneity in refractoriness. The direction of drift depends on the direction of vortex rotation as follows:

$$\mathbf{V}_{dr} = [\omega^* \mathbf{grad}\ \mathbf{v}] \tag{2.4}$$

where ω – vector of the angular velocity of rotation and **grad v**—vector orthogonal to the fibers and directed toward the fiber with a higher velocity.

Figure 4 shows the dependence of velocity V_{dr} on the excitation propagation velocity inhomogeneity. It is seen that when the inhomogeneity changes sign, there is a change in the direction of drift. The velocity V_{dr} grows with increasing inhomogeneity, remaining, however, far below the value observed in the case of equal degree of inhomogeneity in refractoriness. Thus, $V_{dr} \approx 0.1$ at $\Delta V/V_{coll} \approx 0.4$, being about one-fifth as large as the vortex velocity for the corresponding degree of inhomogeneity in refractoriness (cf. Figs 4 and 3).

6. The period of collective waves emitted by a drifting vortex in the direction of its drift, T_f, agrees well with the value of the refractory period of the fiber having a greater refractoriness, R_2. Thus, $T_f = 44.1$ at $R_2 = 47.8$. The period of collective waves propagating in the opposite direction is $T_b = 88.6$, which is also consistent with the value obtained from equation (1.8), $T_b = 85.0$ ($V_{dr} = 0.338$, $V_{coll} = 1.206$).

3. ANALYTICAL ESTIMATION OF THE DRIFT VELOCITY OF A VORTEX IN THE FITZHUGH–NAGUMO TYPE EQUATIONS

The construction of a strict analytical theory of the drift of the vortices in heterogeneous excitable media is very difficult problem which needs to take into consideration a number

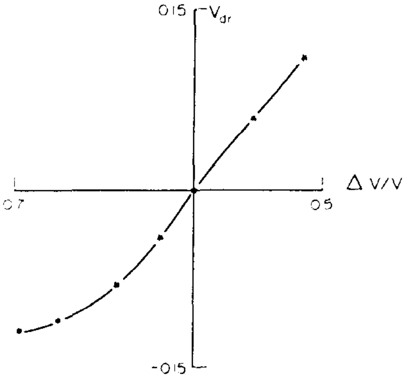

Fig. 4. Drift of a vortex as a function of the degree of inhomogeneity on velocity of waves $(V_{sing1} - V_{sing2})/V_{sing1}$ ($P = 0.15$, $R_1 = R_2 = 35.2$, $r = 0.69$). The velocity was changed by varying the diffusion coefficient: $D = 2$–0.125.

of factors (dispersion relations for wave propagation, influence of the neighboring fiber on the speed of the wave, nonstationarity of the processes, possible drift of the vortices even in homogeneous system of coupled fibers etc.). However, in some cases this problem can be considerably simplified. In fact, for the model (2.1) used for computer simulation the neighboring fibers affects each other in very simple manner. Because of the specific shape of the function $f(E)$ the wave in (2.1) consists of front, back and two regions: $E \equiv 1$ at the plateau, and $E \equiv 0$ after the back of the wave. If we consider a wave with small duration of plateau ($\tau \ll R_1$) than the coupling term $\pm P^*(E_2 - E_1)$ will be equal to $-P^* E_i$ (where i is the number of the fiber) for propagating front of a single wave and 0 for collective wave, as the value of E on the adjacent fiber equals to 0 for single and E_i for collective wave. It is important that the form of these terms do not change in time. So, the chained equations (2.1) unchain and this enables us to perform some analytical estimations and to improve the formula (1.3) for model (2.1). However, even in this case it is difficult to take care for all possible factors. So, here we shall consider the case of high heterogeneities. In particular, the case when the shift of the vortex L (see Fig. 2) is more with respect to the vortex size AB. In this case the vortex motion consists mainly of a lag of the single wave from the collective wave on the interval AC and the dynamics of this process will give us the formula for the drift of the vortex in a system of coupled heterogeneous excitable fibers.

To obtain the value of the velocity we assume that $V_{\text{sing}} = V(l)$, where $V(l)$ is the dispersion relation and l is the distance between the single wave and the collective one. (Note that the axiomatic estimation of the drift of a vortex (Section 1) assumes that the velocity of a single wave, V_{sing}, is constant.)

Let us determine the velocity of drift in this case. As before (Section 1), to do this one needs to find the distance L between points A and C (Fig. 2) where the single wave passes to the second fiber and the time t taken for the single pulse to pass from A to C. Note that during time t the distance between the single and collective waves grows from l_0 (at point A) to l_1 (at point C) and:

$$l_0 = R_1 * V_{\text{coll}} \tag{3.1}$$

$$l_1 = R_2 * V_{\text{coll}}. \tag{3.2}$$

To find time t, consider the dependence of the distance between the single and collective waves on time:

$$\mathrm{d}l/\mathrm{d}t = V_{\text{coll}} - V(l) \tag{3.3}$$

Hence,

$$t = \int_{l_0}^{l_1} \frac{\mathrm{d}l}{V_{\text{coll}} - V(l)} \tag{3.4}$$

where l_0 and l_1 are found from equations (3.1) and (3.2).

Given t, it is easy to find the distance between the points of the successive passage of the single wave to the second fiber (A and C):

$$L = V_{\text{coll}} * t + R_1 * V_{\text{coll}} - R_2 * V_{\text{coll}}. \tag{3.5}$$

Now, to find the velocity of drift it remains only to take into account that the time needed for a single wave to pass the distance between points A and B and back is equal to R_1. Hence,

$$V_{\text{dr}} = L/(t + R_1) \tag{3.6}$$

Therefore, to determine the drift velocity of a vortex one must find L and t. To do this it is necessary to determine the shape of the function $V(l)$. Let us look for the dispersion relation in the class of two-component relaxation models of the reaction-diffusion type. In such models, wave velocities are determined by the value of the slow variable at the impulse front [9]. If $g = 0$ corresponds to the resting state of an excitable medium, then to a first approximation we can write:

$$V(l) = V_1 + V_g'^* g(l), \tag{3.7}$$

where V_1 is the velocity of a single wave at $g = 0$, V_g' is the velocity derivative with respect to the variable g, and $g(l)$ is a value of the slow variable at the wavefront, depending on the distance l between the single and the collective waves. For rather a broad class of excitable media (where equilibrium is represented by a singularity of the saddle type in a complete self-model equation system), the quantity g depends on l (for large l) as follows [10, 11]:

$$g(l) = G^* \exp(-l/\lambda) \tag{3.8}$$

where λ is the eigenvalue characterizing the distance l, on which an e-fold decrease of g is observed, and G is an integration constant.

Considering (3.8), the relation (3.7) can be written as

$$V(l) = V_1 + V_g'^* G^* \exp(-l/\lambda). \tag{3.9}$$

Given the velocity of a single wave at point A, $V(l_0)$, the constant $V_g'^* G$ can be eliminated from (3.9) and the dispersion relation is written as:

$$V(l) = V_1 - (V_1 - V(l_0))^* \exp(-(l - l_0)/\lambda). \tag{3.10}$$

It turns out that in this case the exact value of integral (3.4) can be obtained and the time required for the wave to pass from A to C is:

$$t = \frac{\lambda}{V_{coll} - V_1} {}^*\ln\left(\frac{V_1 - V(l_0) + (V_{coll} - V_1)^* \exp((l_1 - l_0)/\lambda)}{V_{coll} - V(l_0)}\right). \tag{3.11}$$

Thus, by substituting (3.11) into (3.5) and (3.6), we can find the drift velocity of a vortex.

Let us find the drift velocity for the model (2.1). For equation (3.6) to be applicable to model (2.1), one needs to find V_{coll}, V_1, $V(l_0)$, R_1, R_2 and λ. From [5, 10],

$$V_{coll} = 2^*\gamma^* \sqrt{\frac{C}{\gamma^2 + \pi^2}} \tag{3.12}$$

where

$$\gamma = \mathrm{Ln}\,\frac{1 - a}{a}. \tag{3.13}$$

To find V_1 and $V(l_0)$, we use the expression for the velocity of a single wave as a function of the g value at the forward wavefront obtained in [5]:

$$V_{sing} = 2^*\gamma^* \sqrt{\frac{C_n}{\gamma_n^2 + \pi^2}} \tag{3.14}$$

where $\gamma_n = \mathrm{Ln}\, 1 - a_n/a_n$, $C_n = C - P$, $a_n = C^*a + g/C - P$, and also the expression for g at instant of time when a single wave passes to the neighboring fiber:

$$g = P - C^*a. \tag{3.15}$$

To find V_1 from (3.14), we take $g = 0$ and $V(l_0)$ is equal to V_{sing} for g taken from (3.15).

According to [5, 10], the quantities R_1, R_2, and λ can also be expressed through the parameters of model (2.1), namely:

$$\lambda = V_{coll}/\varepsilon_1 \tag{3.16}$$

$$R = T_1 + T_2 + T_3 + T_4 \tag{3.17}$$

where:

$$T_1 = \frac{1}{2^*\gamma} * \frac{\pi^2 + \gamma^2}{C} \tag{3.18}$$

is the duration of the forward wavefront:

$$T_2 = \varepsilon_1^{-1*} \mathrm{Ln} \frac{1 - g^+}{1 - g^-} \tag{3.19}$$

the duration of the excited state;

$$T_3 = T_1 \tag{3.20}$$

the duration of the back wavefront;

$$T_4 = \varepsilon_1^{-1*} \mathrm{Ln} \frac{g^-}{g^+} \tag{3.21}$$

the duration of the refractory tail.

Here g^+ is the value of g at the front of an excitation wave found from (3.15), and g^- is its value at the back,

$$g^{-1} = C^*(1 - 2^*a) \tag{3.22}$$

The value of γ is found from the relation (3.13). It can see from equations (3.21), (3.19) and (3.14) that the refractoriness of the fibers depends from coupling coefficient.

Substituting the expressions for R, λ, V_1, $V(l_0)$ and V_{coll} into (3.1), (3.2), (3.11) and then into (3.5) and (3.6), we find the velocity of drift of a vortex, V_{dr}.

Figure 5 presents plots of V_{dr} as a function of the degree of inhomogeneity $\Delta R/R$ obtained from numerical experiments, from analytical estimate (3.6), and from axiomatic

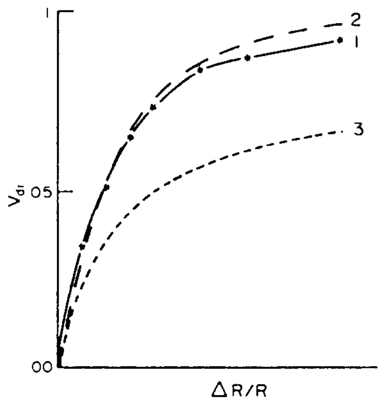

Fig. 5. Dependence of the vortex drift velocity on the degree of inhomogeneity on refractoriness $\Delta R/R_1$: 1, in numerical experiment; 2, as obtained from analytical estimate from equation (3.6); 3, as obtained from equation of the axiomatic approach (1.3). $R_1 = 41.4$, $P = 0.15$, $\tau = 6.9$.

estimate (1.3) with the velocities of the single and collective waves and the refractoriness of fibers given from analytical equations (3.12), (3.14), (3.17)–(3.21). It is seen that the discrepancy between the analytical estimate and the experimental curves does not exceed 5% of the drift velocity. It is also seen that the analytical estimate is better than the previous axiomatic approach.

Analogous plots are presented in Fig. 6 for the drift velocity as a function of the coupling coefficient in a system inhomogeneous with respect to refractoriness. There is fairly good agreement between the plots for large values of the coupling coefficient, P. As P decreases, the agreement becomes worse, the theoretical values for V_{dr} being above those obtained experimentally. This seems to be due to the fact that the vortex size (distance between A and B) increases when the coupling between the fibers weakens, and the fact that the distance between the single and collective waves increases when the former moves from point B to point A, which we ignored in the derivation of equation (3.6), should be taken into consideration.

DISCUSSION

This work has shown that the presence in an excitable medium of an inhomogeneity with respect to the propagation velocity leads to the drift of a vortex. The drift mechanism in this case appears to be as follows.

The refractoriness of a medium is not completely fixed, as was supposed in Section 1, but it depends on the strength of the applied stimulus. This effect is well known in physiology as the phenomenon of relative refractoriness of a medium and is usually described by a strength–duration curve [12]. In a medium having an inhomogeneity with respect to the propagation velocity, the wave length on one of the fibers is greater than on the other. As a result, a wave propagating along the first fiber produces a stronger stimulating effect on the second fiber than in the opposite case. Therefore, the actual refractoriness of the second fiber is lower than that of the first one, and this gives rise to the drift of the vortex.

We studied here the drift phenomenon in an excitable medium with a stepped inhomogeneity in refractoriness. Such inhomogeneities are, undoubtedly, of great interest in biological excitable media, in the first place, in cardiac tissue, where the possibility of this type of vortex generation was demonstrated [13–15].

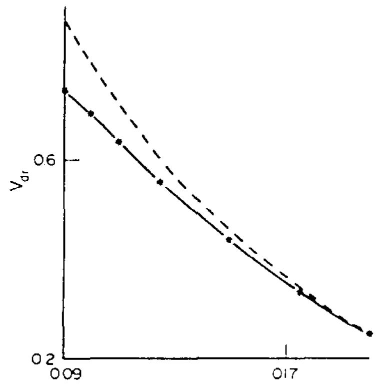

Fig. 6. Dependence of the vortex drift velocity on the coupling coefficient. Solid line, numerical experiments; broken line, analytical data obtained from equation (3.6); $\varepsilon_1 = 10$ for fiber 1 ($R = 41.4$ for $P = 0.15$), $\varepsilon_1 = 14$ for fiber 2 ($R = 51.6$ for $P = 0.15$), $\tau = 6.9$.

The results obtained suggest that vortices caused by a stepped inhomogeneity must drift along its border and the direction of the drift velocity being given by equation (1.4), i.e. being dependent on the directions of vortex rotation and of inhomogeneity orientation. There is another interesting fact that comes from equation (1.4). If there are two vortices generated in a medium by the rhythm transformation mechanism, than, as shown in [6], they will have opposite sense of rotation. In this case, besides opposite rotation, they will have a special orientation with respect to the inhomogeneity [6]. As a result of equation (1.4), such vortices must approach each other. They may then collide and annihilate. This process may underlay the phenomenon of the finite life-time of a vortex in cardiac tissue [14].

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