National Biofilms Innovation Centre

MODELLING PATTERN FORMATION IN HETEROGENEOUS BACTERIAL POPULATIONS

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In this work we model the very early stages of biofilm formation when bacteria are loosely attached to a surface such that they are still motile. We consider two interacting bacterial populations which proliferate and compete for resources. First, we consider formation of travelling wave solutions in a system of two interacting bacterial populations. Then we introduce chemotactic activity for one of the populations in response to a chemical produced by the second population and analyse formation of Turing patterns in this system.

INTRODUCTION

- Motile bacteria can form spatial-temporal patterns due to taxis to external or internal signals. Understanding how these patterns are initiated is important in biofilm prevention since surface attachment, reproduction and formation of colonies is the first step in the formation of biofilms.

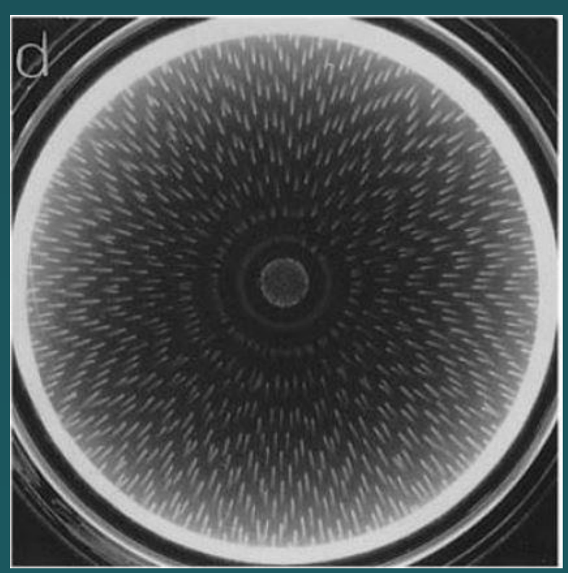

***Fig1:** Pattern formed by E. Coli bacteria. It is initially placed on a small spot in the center of the petri dish spreads out and covers the entire surface with a stationary spotted pattern. [1, Ch5]*

NONDIMENSIONAL MATHEMATICAL MODEL

$$\begin{cases} \dfrac{\partial n}{\partial t} = D_1 \dfrac{\partial^2 n}{\partial x^2} + \chi \dfrac{\partial}{\partial x}\left(n \dfrac{\partial c}{\partial x}\right) + r_1 n(1 - n - b_1 v) \\ \dfrac{\partial v}{\partial t} = D_2 \dfrac{\partial^2 v}{\partial x^2} + r_2 v(1 - v - b_2 n) \\ \dfrac{\partial c}{\partial t} = \dfrac{\partial^2 c}{\partial x^2} + v - c \end{cases}$$

- n, v, c represent two competing bacterial species and a chemical which can diffuse with ratios D_1, D_2, reproduce with ratios r_1, r_2 and compete with ratios b_1, b_2 respectively.

STABILITY IN THE WELL-MIXED SYSTEM

- The system we consider has four steady states (n^*, v^*, c^*):

$$(0,0,0); (1,0,0); (0,1,1); \left(\frac{b_1 - 1}{b_1 b_2 - 1}, \frac{b_2 - 1}{b_1 b_2 - 1}, \frac{b_2 - 1}{b_1 b_2 - 1}\right)$$

- Stability of these steady states is important since travelling wavefronts make the transition between stable and unstable steady state; and stationary spatial patterns appear when the stable steady state in the well-mixed system becomes unstable in presence of diffusion and chemotaxis

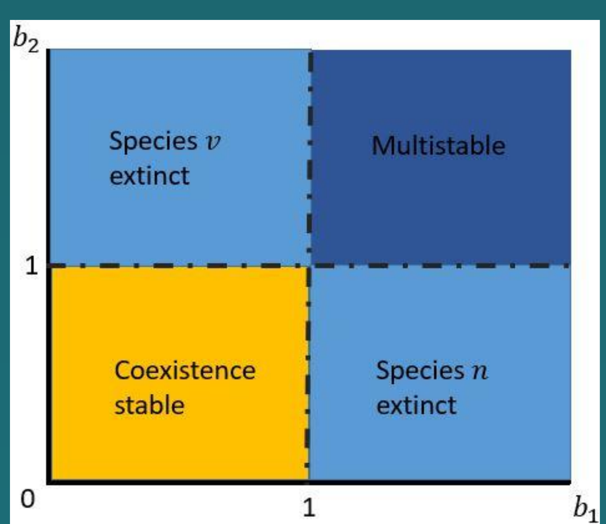

***Fig2:** Regions in which different steady states are stable depending on the competition ratios b_1, b_2. $(0,0,0)$ always unstable. [2]*

TRAVELLING WAVEFRONTS

- Here, we consider a system of two interacting bacterial populations in which there is no chemical being produced and hence no chemotaxis, $c = 0, \chi = 0$, we have investigated the transition between different steady states as travelling wavefronts with speed s. [3, 4]
- Wavefronts move with different speeds in the three simulations presented because they are transitioning from different steady states. The speeds depend on the parameters set, but there are different speed requirements for different steady states.

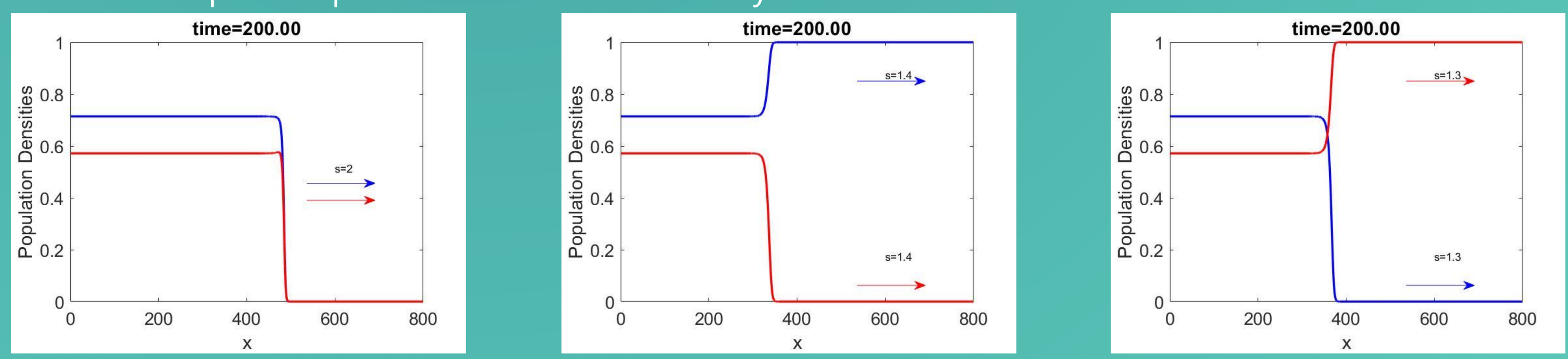

***Fig3:** In the first panel, wavefronts transtition from $(0,0)$ to coexistence. In the second panel, they transition from $(1,0)$ to coexistence; and in the third panel they transition from $(0,1)$ to coexistence. Set parameters are $D_1 = D_2 = 1, r_1 = r_2 = 1, b_1 = 0.5, b_2 = 0.6$. Simulations are set such that in all panels there is a small region $x\epsilon(0,100)$ in which concentrations of the bacteria are such that $(n, v) = (1,1)$ and different anywhere else. In the region $x\epsilon(100,800)$, $(n, v) = (0,0)$ in the first panel, $(n, v) = (1,0)$ in the second panel and $(n, v) = (0,1)$ in the third panel.*

TURING PATTERNS

- In this section we are looking at the formation of spatial patterns as a breakdown of stability due to changes in the parameters that characterize the system. [5]
- First, we have set all parameters apart from competition (b_1, b_2) and obtained that if these values are within the loop, then we expect to see a breakdown of stability and hence Turing patterns. Then, we have set $b_1 = b_2 = b$ and looked at the relationship between competition and chemotaxis χ. Clearly, as the chemotactic sensitivity increases, competition decreases. Also, there is a vertical asymptote at $\chi = 7$, such that no matter how strong competition between species is, the system will relax back to homogeneous steady state when perturbed.

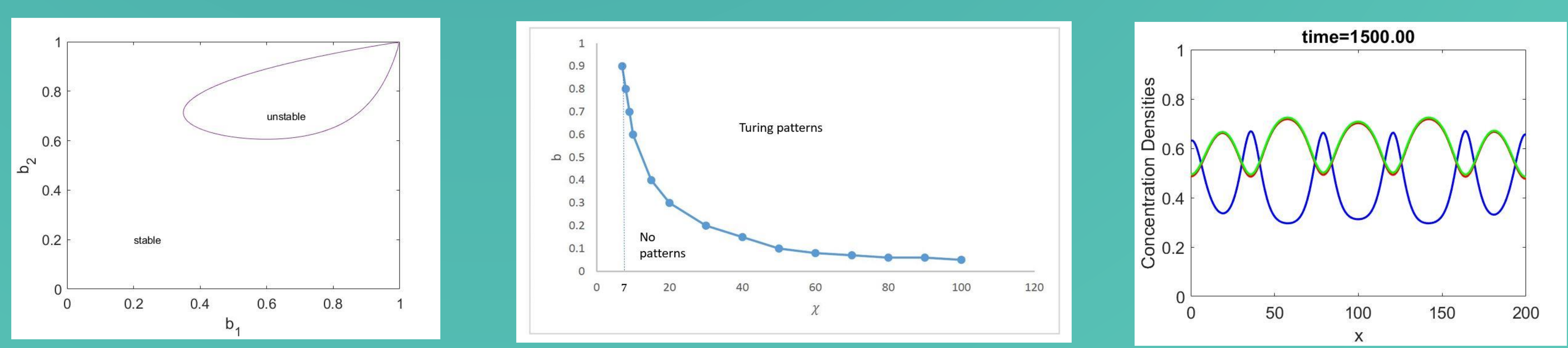

***Fig4:** In the first panel, inside the loop is the largest Turing space (b_1, b_2) obtained for set parameters $D_1 = D_2 = 1, r_1 = r_2 = 0.1$ and $\chi = 10$. In the second panel we look at the relationship between competition and chemotaxis. In the third panel, we simulate a Turing pattern for $\chi = 7$, $b_1 = b_2 = b = 0.9$.*

DISCUSSION

We have introduced a model for two interacting bacterial species such that one of the species produces a chemotactic agent for the other species. We have shown that different spatial patterns such as travelling wavefronts and Turing patterns can be observed in this system. We have presented numerical simulations of the travelling wavefronts between different steady states. These are travelling with different speeds depending on the steady state we are transitioning from. Numerical speeds are presented, but these are matching the analytical speeds we have obtained. We have obtained domains for the competition effects that lead to a breakdown of stability and hence, Turing patterns. There is a domain $\chi < 7$, in which we cannot have patterns no matter how strong competition is.

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