PHYS393: Statistical and Low Temperature Physics

Tutorial 3

These questions cover Parts 4 and 5 of the lecture course.

The marks for each question are shown in square brackets. Questions shown as *"Optional additional question"* will not contribute to your overall marks.

Please hand in your solutions to the student office by **4 pm on Monday 24 November**.

The following constants may be useful:

Planck's constant,	h	$=$	6.63×10^{-34} Js
Boltzmann constant,	k_B	$=$	1.38×10^{-23} J/K
Speed of light,	c	$=$	3.00×10^{8} m/s
Electron mass,	m_e	$=$	9.11×10^{-31} kg

1. A system is composed of N conduction electrons that move freely within a block of metal of volume V.

   (a) Given that the number of states $g(k)\,dk$ with wave number k in the range k to $k+dk$ is:
   $$g(k)\,dk = \frac{2V}{(2\pi)^3}4\pi k^2\,dk,$$
   derive an expression for the number of states with energy in the range ε to $\varepsilon+d\varepsilon$. **[2]**

   (b) The probability that a state with energy ε will be occupied by an electron is $f(\varepsilon)$. Sketch graphs of $f(\varepsilon)$ versus ε: (i) for temperature $T=0$; (ii) for temperature T such that $0<T<T_F$, where T_F is the Fermi temperature. Indicate the Fermi energy ε_F. **[3]**

   (c) Show that the Fermi energy ε_F is given by:
   $$\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}}.$$
   [2]

   (d) Metallic silver has a molar volume of $10.27\times10^{-6}\,\mathrm{m}^3$, and each atom contributes one electron to the conduction band. Evaluate the Fermi energy ε_F for silver. **[2]**

   (e) At temperature T, the total energy U of the electrons can be written:
   $$U = \frac{3}{5}N\varepsilon_F\left[1+\frac{5\pi^2}{12}\left(\frac{k_BT}{\varepsilon_F}\right)^2\right]$$
   Evaluate the electron heat capacity C_V for a molar quantity of silver at a temperature of 5 K. **[3]**

2. (a) Explain what is meant by “black body radiation”.

[2]

(b) The density of states $g(k)$ in terms of wave number k for quantised electromagnetic waves in a cavity of volume V is written as:

$$g(k)\,dk = \frac{2V}{(2\pi)^3} 4\pi k^2\,dk.$$

Show that this density of states can be written in terms of the frequency ν of the waves as:

$$g(\nu)\,d\nu = \frac{8\pi V}{c^3}\nu^2\,d\nu,$$

where c is the speed of light.

[2]

(c) The energy contained in the frequency interval ν to $\nu + d\nu$ of the radiation is given by:

$$u(\nu)\,d\nu = \frac{8\pi V}{c^3}\nu^2\,d\nu \cdot h\nu \cdot \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}.$$

Explain the physical significance of the factors $h\nu$ and $\left[\exp(h\nu/k_B T) - 1\right]^{-1}$ in this expression.

[2]

(d) Deduce the limiting values of $u(\nu)$ as $\nu \to 0$ and as $\nu \to \infty$.

[4]

(e) Sketch the distribution of $u(\nu)$ versus ν for two different temperatures. Mark on your graph which curve corresponds to the higher temperature.

[3]

(f) *Optional additional question:* Show that the energy density U/V in the cavity is given by:

$$\frac{U}{V} = \frac{8\pi^5}{15}\frac{(k_B T)^4}{(hc)^3}.$$

(g) *Optional additional question:* Evaluate the energy density U/V at a temperature $T = 1000\,\mathrm{K}$.

(h) *Optional additional question:* Calculate the temperature at which the energy density is 10 times greater than it is at $T = 1000\,\mathrm{K}$.

You are given that:

$$\int_0^\infty \frac{y^3\,dy}{e^y - 1} = \frac{\pi^4}{15}.$$