PHYS393: Statistical and Low Temperature Physics

Tutorial 2

These questions cover Part 3 of the lecture course.

The marks for each question are shown in square brackets. Questions shown as *"Optional additional question"* will not contribute to your overall marks.

Please hand in your solutions to the student office by **4 pm on Monday 3 November**.

Consider a large number N of microscopic particles, confined within a *2-dimensional* square plane of side length L, but free to move within that plane. The density of the "gas" of particles is low, so that particles collide only occasionally with each other or with the sides of the plane. The particles have no internal structure.

1. Derive an expression for the number of states $g(k)\,dk$ in which the wave vector of a particle lies between k and $k + dk$. State the regime in which your expression for $g(k)\,dk$ is valid.

   [4]

2. Using your expression for $g(k)\,dk$, derive an expression for the number of states $g(\varepsilon)\,d\varepsilon$ in which the energy of a particle lies between ε and $\varepsilon + d\varepsilon$. Express your answer in terms of the parameter θ given by:

   $$\theta = \frac{\hbar^2 \pi^2}{2mk_B A},$$

   where $\hbar$ is Planck's constant divided by 2π, m is the mass of a particle, k_B is Boltzmann's constant, and $A = L^2$ is the area of the plane.

   [4]

3. By integrating over the density of states $g(\varepsilon)$, derive an expression for the partition function Z of the system. Express Z in terms of the temperature T and the parameter θ.

   [3]

4. Using the partition function, derive an expression for the total energy of the particles as a function of temperature. Hence, write down an expression for the heat capacity of the system at constant area A.

   [4]

5. Write down an expression for the Helmholtz free energy, F. Using this expression, find the entropy S of the system as a function of temperature.

   [3]

6. Write an expression for dF, the change in free energy, in terms of changes dT and dA in temperature and area, respectively. Hence find an expression for the "pressure", i.e. the force per unit length of the enclosure from the particles. (You may assume that the first law of thermodynamics takes the form $dU = T\,dS - p\,dA$.)

[4]

7. Find the energy distribution $n(\varepsilon)$, and the velocity distribution, $n(v)$ of the particles.

[3]

8. *Optional additional question:* Find expressions for the most likely velocity, the mean velocity, and the root mean square velocity; sketch the velocity distribution, and indicate these quantities on your plot.

9. *Optional additional question:* Without detailed calculation, explain how you would expect the heat capacity of the gas to behave if the particles, instead of having no internal structure, were replaced with diatomic molecules.