PHYS393: Statistical and Low Temperature Physics

Tutorial 1

These questions cover Parts 1 and 2 of the lecture course.

The marks for each question are shown in square brackets. Questions shown as *"Optional additional question"* will not contribute to your overall marks.

Please hand in your solutions to the student office by **4 pm on Monday 13 October**.

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1. An unbiased coin is tossed N times.

   (a) Write down an expression for the total number of different sequences of heads and tails, Ω.

   [2]

   (b) Write an expression for the number of different sequences that contain H heads and T tails, $t(H, T)$.

   [2]

   (c) Show that if N is large enough for Stirling's approximation to be valid, then $t(N/2, N/2) \approx \Omega$. Comment on the significance of this result.
   Hint: Evaluate $\ln t(N/2, N/2)$*, and compare with* $\ln \Omega$.

   [4]

2. A model thermodynamic assembly has single-particle states with energies 0, ϵ, 2ϵ, 3ϵ, 4ϵ, 5ϵ, 6ϵ... The assembly has four distinguishable particles and a total energy of 6ϵ.

   (a) Draw a table showing the number of particles in each energy state and the number of microstates in each of the nine possible distributions.
   Hint: you can use the formula $t_{\{n_i\}} = N!/n_1!n_2!\ldots$ *to calculate the number of microstates in each distribution.*

   [4]

   (b) Find the total number of accessible microstates.

   [1]

   (c) Find the mean populations of each of the energy states. Comment on whether the mean distribution has the shape you would expect.

   [5]

3. The single-particle energy states in a system of distinguishable particles have energies 0, ϵ and 2ϵ. The middle level is doubly degenerate (two quantum states have the same energy) and the other levels are singly degenerate.

   (a) Write down an expression for the partition function Z as a function of the temperature T.

   [2]

   (b) By summing over energy states, find an expression for the total energy U for a collection of N particles, as a function of temperature T. Show that the result you obtain in this way is the same as that obtained from:

   $$U = N\frac{d \ln Z}{d\beta},$$

   where $\beta = -1/k_B T$, and k_B is Boltzmann's constant. Find the limiting energy as $T \to 0$, and as $T \to \infty$; explain these limits in physical terms.

   [5]

   (c) *Optional additional question:* Derive an expression for the heat capacity at constant volume, C_V.

   (d) *Optional additional question:* Starting from the Helmholtz free energy (expressed in terms of the partition function), find an expression for the entropy, S, of the system as a function of temperature. Find limiting expressions for S when $T \to \infty$ and when $T \to 0$. Sketch a plot showing the variation of entropy with temperature.