Tutorial 3

These questions cover Parts 6 and 7 of the Lecture Course.

Please give your solutions to your tutor by noon on Wednesday 26th March.

Question 1

a) A transmission line has capacitance per unit length C and inductance per unit length L. Show that the voltage between the conductors in the transmission line satisfies the equation:

$$\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}$$

b) With reference to the peak current and peak voltage in the transmission line, explain what is meant by the "characteristic impedance", Z. Show that the characteristic impedance is given by:

$$Z = \sqrt{\frac{L}{C}}$$

c) A coaxial cable is designed to have a characteristic impedance of 75 Ω. If the inner conductor has a diameter of 2 mm, calculate the diameter of the outer conductor. (Assume that the conductors are separated by a non-magnetic dielectric with relative permittivity 1).

A 75 Ω cable is used to connect a signal source with characteristic impedance 75 Ω to a load with characteristic impedance 50 Ω. Calculate the fraction of *power* reflected from the load.

Question 2 *(This question is from the 2007 exam)*

a) Write down expressions for the electric field $\vec{E}$ and magnetic field $\vec{B}$ in terms of the scalar potential ϕ and vector potential $\vec{A}$.

b) The potentials ϕ' and $\vec{A}'$ are obtained from the potentials ϕ and $\vec{A}$ by:

$$\phi' = \phi + \dot{\psi}$$

$$\vec{A}' = \vec{A} - \nabla \psi$$

where ψ is a function of time and position. Show that the electric and magnetic fields derived from the potentials ϕ' and $\vec{A}'$ are the same as the fields derived from the potentials ϕ and $\vec{A}$.

c) Suppose that the potentials ϕ' and $\vec{A}'$ satisfy the equation:

$$\nabla \cdot \vec{A}' + \mu\varepsilon \dot{\phi}' = f$$

for some function f of time and position. Show that, if ψ satisfies the equation:

$$\nabla^2 \psi - \mu\varepsilon \ddot{\psi} = -f$$

then the potentials ϕ and $\vec{A}$ satisfy the Lorenz gauge condition:

$$\nabla \cdot \vec{A} + \mu\varepsilon \dot{\phi} = 0 .$$