Tutorial 2

These questions cover Parts 3, 4 and 5 of the Lecture Course.

Please give your solutions to your tutor by noon on Wednesday 5th March.

Question 1 *(This is an abbreviated version of a question from the 2008 exam)*

a) Write down Maxwell's equations in an ohmic conductor, and derive from them the wave equation for the electric field inside the conductor.

b) Write down a solution for the wave equation in an ohmic conductor, representing a plane wave of frequency ω. Find expressions for the real and imaginary parts of the wave vector in terms of the frequency of the wave and the properties of the conductor. Explain the physical significance of the real and imaginary parts of the wave vector.

c) Explain physically why an electromagnetic wave is attenuated when it propagates in a conducting medium.

Question 2 *(This question is from the 2006 exam)*

An unpolarised plane electromagnetic wave in free space is incident normally on a slab of glass with refractive index $n = 1.5$. Estimate the fraction of incident power transmitted into the glass.

Question 3 *(This is an abbreviated version of a question from the 2007 exam)*

In free space the electric field $\vec{E}$ satisfies the wave equation:

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

a) Show that the wave equation is satisfied by an electric field with components:

$$E_x = E_{x0} \cos k_x x \sin k_y y \exp\ j(\omega t - k_z z)$$

$$E_y = E_{y0} \sin k_x x \cos k_y y \exp\ j(\omega t - k_z z)$$

$$E_z = -jE_{z0} \sin k_x x \sin k_y y \exp\ j(\omega t - k_z z)$$

if the frequency ω of the field satisfies:

$$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2$$

b) Using Maxwell's equation $\nabla \cdot \vec{E} = 0$, derive a constraint on the field amplitudes E_{x0}, E_{y0}, E_{z0}, and the wave numbers k_x, k_y, k_z.

<u>Question 3</u> *(Continued)*

c) Show that, if the above electric field exists in a rectangular waveguide with vacuum in the interior and perfectly conducting walls at $x = 0$, $x = a_x$, $y = 0$ and $y = a_y$, the boundary conditions impose the constraints:

$$k_x = \frac{n_x \pi}{a_x} \qquad k_y = \frac{n_y \pi}{a_y}$$

for integers n_x and n_y.

d) The group velocity of a wave in a waveguide is given by:

$$v_g = \frac{d\omega}{dk_z}.$$

Explain the physical significance of the group velocity.

Show that, for the electric field given above, the group velocity in the rectangular waveguide can be expressed in terms of the mode numbers n_x and n_y by:

$$v_g = c\sqrt{1 - \frac{\pi^2 c^2}{\omega^2}\left(\frac{n_x^2}{a_x^2} + \frac{n_y^2}{a_y^2}\right)}.$$

e) From the above expression for the group velocity, derive an expression for ω_c, the frequency of the lowest frequency wave that can be propagated in the waveguide for given mode numbers n_x and n_y.

f) Sketch a plot showing the variation of the group velocity v_g with frequency ω for frequencies above ω_c, indicating the behaviour of the group velocity in the limit $\omega \to \omega_c$ and in the limit $\omega \to \infty$.