Tutorial 1

These questions cover Parts 1 and 2 of the Lecture Course.

Please give your solutions to your tutor by noon on Wednesday 12th February.

Question 1

a) Explain what is meant by the *displacement current*.

b) Starting from the appropriate Maxwell's equations, and using the appropriate integral theorems, derive the continuity equation:

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}.$$

What is the physical significance of this equation?

Question 2

A parallel plate capacitor is connected to a cell so that an electric current I flows through the connecting wires. By using the appropriate Maxwell's equation, and applying Stokes' theorem to a circular disc centred on a wire connected to one of the plates of the capacitor, show that the magnitude of magnetic field B at a distance r from the wire is given by:

$$B = \frac{\mu}{2\pi}\frac{I}{r},$$

where μ is the permeability of the medium surrounding the wire.

Show that the same result is obtained if Stokes' theorem is applied to a surface S with the same boundary, but formed so that it passes between the plates of the capacitor (i.e. no conduction current passes through the surface S).

Question 3 *(This question is from the 2008 Exam)*

When an electromagnetic wave passes through a dielectric, the equation of motion for an electron in the material can be written:

$$\ddot{x} + \Gamma\dot{x} + \omega_0^2 x = \frac{e}{m}E_0 e^{j\omega t}$$

where ω is the frequency of the wave, and E_0 is the amplitude of the electric field in the wave. Explain the physical origin of each term in the equation.

Show that a solution to the equation of motion is given by:

$$x = \frac{(e/m)E_0 e^{j\omega t}}{(\omega_0^2 - \omega^2) + j\omega\Gamma}.$$

The polarisation P of the dielectric in an electric field E is given by:

$$P = Nex = \chi_e \varepsilon_0 E.$$

where N is the number of electrons per unit volume of the material.

Show that the electric susceptibility χ_e can be written:

$$\chi_e = \frac{Ne^2}{m\varepsilon_0}\frac{1}{(\omega_0^2 - \omega^2) + j\omega\Gamma}.$$

Write down the relationships between

(i) the relative permittivity ε_r and the susceptibility χ_e;

(ii) the refractive index n and the relative permittivity ε_r.

Hence show that the refractive index is related to the susceptibility by:

$$n = \sqrt{1 + \chi_e}.$$

Assuming that $|\chi_e| \ll 1$, expand the refractive index as a Taylor series to first order in the susceptibility. Hence find the real and imaginary parts of the refractive index, $n = n_1 - jn_2$, as functions of the wave frequency ω.

Sketch a plot of n_1 and n_2 as a function of the wave frequency ω. Indicate on your plot regions of normal and anomalous dispersion. Describe how the shape of the plot depends on the parameters ω_0 and Γ.