Cockcroft Lectures: Linear Dynamics

Problem Set 7

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1. The energy deviation δ for a particle of rest mass m is defined by:

$$\delta = \frac{\gamma m c}{P_0} - \frac{1}{\beta_0} \tag{1}$$

where γ is the relativistic factor for the particle, P_0 is the reference momentum, and $\beta_0 c$ is the velocity of a particle with rest mass m and momentum equal to the reference momentum. Show that:

$$\beta\gamma = \beta_0\gamma_0\sqrt{1 + \frac{2\delta}{\beta_0} + \delta^2} \tag{2}$$

where βc is the velocity of a particle with energy deviation δ, and γ_0 is the relativistic factor for a particle with rest mass m and momentum equal to the reference momentum.

2. Consider a particle moving in a horizontal plane in a uniform vertical magnetic field. The region of the field is large enough for the trajectory of the particle to describe a complete circle. Using the result from Problem 1, show that the circumference C of the trajectory as a function of the energy deviation δ is given by:

$$\frac{\Delta C}{C_0} = \sqrt{1 + \frac{2\delta}{\beta_0} + \delta^2} - 1 \tag{3}$$

where

$$\Delta C = C - C_0 \tag{4}$$

and C_0 is the circumference of the trajectory when $\delta = 0$.

3. Is the particle described in Problem 2 above, below or at transition? Explain, by considering the period of the motion of the particle as a function of the energy of the particle.