Cockcroft Lectures: Linear Dynamics

Problem Set 5

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1. Consider the motion of a particle moving in the horizontal plane, passing through a thin horizontally-focusing quadrupole. Starting from the Hamiltonian expanded to second order in the transverse variables, while keeping the full dependence on the energy deviation δ, calculate the focal length of a *thin* quadrupole as a function of δ.

2. Using Newtonian mechanics and the Lorentz force on a charged particle in a magnetic field, calculate the focal length of a *thin* quadrupole as a function of the total normalised momentum p of the particle. (The normalised momentum is given by $p = P/P_0$, where P is the total mechanical momentum, and P_0 is the reference momentum. Assume that $|Kx_0/p| \ll 1$, where K is the integrated normalised strength of the quadrupole, and x_0 is the horizontal distance from the optical axis at which a particle passes through the quadrupole).

3. Show that in the ultrarelativistic limit $\beta \to 1$, your expressions for the focal length calculated using Hamiltonian mechanics (Problem 1) and using Newtonian mechanics (Problem 2) converge. Comment on the origin of the dependence of focal length on the energy deviation in each calculation.

4. (Bonus question for ambitious students!) A particle approaches a thin quadrupole parallel to the optical axis but horizontally offset from it. Using the Hamiltonian formulation, calculate the change in longitudinal coordinate z of the particle between the quadrupole and the focal point. Compare your result with the quantity you would expect from geometric arguments.