Cockcroft Lectures: Linear Dynamics
Problem Set 4

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1. Write down the 4×4 submatrix for the transverse components of the transfer matrix for a quadrupole of normalised field strength k_1 and length L. Find the form of this 4×4 matrix in the limit $L \to 0$, where:

$$\lim_{L \to 0} k_1 L = K \tag{1}$$

   and K is a finite constant. (This is the *thin lens* approximation: K is the *integrated strength* of the quadrupole).

2. A beamline consists of a thin quadrupole of integrated strength K, a drift of length $1/\sqrt{2}K$, a thin quadrupole of integrated strength $-2K$, a second drift of same length as the first, and a final thin quadrupole of the same integrated strength as the first quadrupole. The beamline is illustrated in Fig. 1.

   (a) By multiplying 2×2 submatrices for the horizontal component of the motion in each of the elements along the beamline, find the 2×2 transfer matrix for the horizontal motion in the entire beamline.

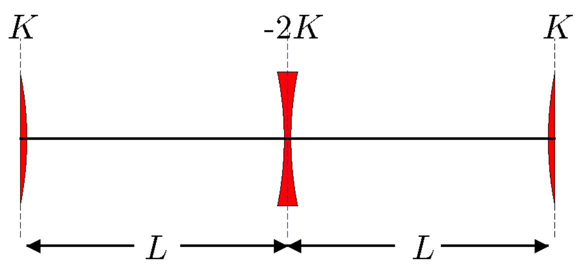

Figure 1: Beamline consisting of a thin quadrupole, a drift, a second thin quadrupole, a second drift, and a third and final thin quadrupole.

(b) Evaluate the 2×2 transfer matrix for the horizontal motion in the beamline in the case:

$$K = 1 + \sqrt{2} \tag{2}$$

(c) Now consider a beamline consisting of the shorter beamline shown in Fig. 1 repeated an infinite number of times. Show that horizontal motion of a particle moving along the infinite beamline must be stable and periodic. What is the period of the motion? Without any additional algebra, describe in general terms the vertical motion of a particle moving along the infinite beamline.