Cockcroft Lectures: Linear Dynamics

Problem Set 3

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1. The *normalised* strength of a normal multipole component of a magnetic field is often specified by the "$k-$value", defined by:

$$k_n = \frac{q}{P_0} \frac{1}{n!} \left. \frac{\partial^n B_y}{\partial x^n} \right|_{x=y=0} \tag{1}$$

where q is the charge on the reference particle, P_0 is the reference momentum, and the derivative is evaluated at $x = y = 0$. Similarly, the *normalised* strength of a skew multipole component of a magnetic field is specified by a value k_{ns}, where:

$$k_{ns} = \frac{q}{P_0} \frac{1}{n!} \left. \frac{\partial^n B_x}{\partial x^n} \right|_{x=y=0} \tag{2}$$

Find the relationships between the values of k_n and k_{ns} for a multipole field, and the values of the coefficients a_n and b_n in the multipole expansion:

$$B_y + iB_x = \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x + iy}{r_0} \right)^{n-1} \tag{3}$$

(Note: the value of n in one representation may be different from the value of n in the other representation!)

2. A particle of charge q moves in a uniform vertical magnetic field of strength B_0.

   (a) Write down the Hamiltonian for the motion of the charged particle expanded to second order in the usual accelerator (normalised) variables and using path length s as the independent variable. Assume that the curvature h of the reference trajectory is equal to the normalised dipole field strength k_0:

$$k_0 = \frac{q}{P_0} B_0 \tag{4}$$

   where P_0 is the reference momentum.

(b) Write down the equations of motion (Hamilton's equations) for the charged particle moving in this field.

(c) Find the solutions to the equations of motion in the case that the vertical momentum is zero. Hence find the period of horizontal oscillations around the reference trajectory for a particle initially close to the reference trajectory. Express the period of the motion in terms of the circumference of the reference trajectory, and explain your answer physically.