Cockcroft Lectures: Linear Dynamics
Problem Set 2

A. Wolski

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A certain magnetic field is given by the vector potential:

$$A_x = 0, \qquad A_y = 0, \qquad A_z = -B_0 x \tag{1}$$

1. Write down the magnetic field derived from the potential (1).

2. Assuming zero electric potential, write down the Hamiltonian for a relativistic particle of charge q and mass m moving in the field derived from the potential (1), and write down the equations of motion for the particle following from this Hamiltonian. (Assume that the Hamiltonian can be identified with the total energy E of the particle).

3. In the case $y = p_y = 0$, show that the particle moves in a circle in the $x - z$ plane, and find expressions for the radius of the circle and the frequency of revolution in terms of the initial conditions:

$$x = x_0, \qquad p_x = p_{x0}, \qquad p_z = p_{z0} \tag{2}$$

   at $t = 0$, and the particle's energy, E. Compare your expressions for the radius and revolution frequency to those derived by considering the motion of the charged particle in the $x - z$ plane in Newtonian mechanics, subject to the Lorentz force.