CERN Accelerator School

Low Emittance Machines

Tutorial Problems

Useful physical constants:

$c = 2.998\times10^{8}$ m/s
$C_\gamma = 8.846\times10^{-5}$ m/GeV^3
$C_q = 3.832\times10^{-13}$ m

Lecture 1 Problem

1. The DIAMOND storage ring has a circumference of 560 m, and contains 48 dipole magnets (without quadrupole gradient), each of length 1 meter. The beam energy is 3 GeV. Calculate:

   a. the bending radius of each dipole;

   b. the second and third synchrotron radiation integrals;

   c. the energy loss of each particle per turn through the ring;

   d. the horizontal, vertical and longitudinal damping times;

   e. the equilibrium energy spread.

Lecture 2 Problem

2. a. Show that, for small phase advance μ_x in a FODO cell, the natural emittance of a FODO lattice is given approximately by:

$$\varepsilon_0 \approx 8C_q\gamma^2\left(\frac{\theta}{\mu_x}\right)^3$$

   where θ is the bending angle of one dipole in the cell, and γ is the relativistic factor.

   b. Given data on a number of storage ring lattices:

Storage ring	Lattice type	Beam energy	Number of dipoles
SRS	72° FODO	2 GeV	16
SRS (HBL)	140° FODO	2 GeV	16
APS	DBA	7 GeV	80
DIAMOND	DBA	3 GeV	48
ALS	TBA	1.9 GeV	36

   estimate the natural emittance for each ring, assuming that the lattice is tuned for the minimum emittance in each case (with zero dispersion in the straight sections of the achromats). *Note: use the small phase advance approximation for the FODO lattices – even though this may not be very accurate in these cases!*

   Explain why the emittances for the achromat lattices are likely to be somewhat larger in practice than the values you have calculated.

Lecture 3 Problem

3. Show that, in one degree of freedom, the eigenvalues of $\Sigma \cdot S$ are $\pm i\varepsilon_x$, where Σ is the "sigma matrix" of second-order moments of the beam distribution, S is the 2×2 antisymmetric matrix defined in the lecture, and ε_x is the beam emittance.

Optional Extra Problem: ILC Damping Rings Case Study

4. The damping rings in the ILC serve the purpose of accepting large-emittance electron or positron beams from the particle sources, and producing low-emittance, highly stable beams for acceleration in the linacs and collision at the interaction point. Each beam is stored for the time between machine pulses, which is 200 ms in the case of ILC.

   The damping rings are synchrotrons, similar to the storage rings in third generation light sources, but rather larger. Some of the parameter specifications for the damping rings are as follows:

Circumference	6.6 km
Energy	5 GeV
Injected emittance (x and y)	1 μm
Extracted horizontal emittance	0.8 nm
Extracted vertical emittance	2 pm
Equilibrium vertical emittance	1.4 pm
Maximum extracted energy spread	0.13%
Beam store time	200 ms
Lattice type	TME
Number of dipoles	120
Dipole length	6 m

   a. Calculate the transverse damping times required to achieve the extracted emittances starting with the specified injected emittances, in the given store time.

   b. Estimate (i) the damping times, and (ii) the natural emittance that would be achieved in the lattice without any damping wiggler (i.e. with the only synchrotron radiation energy loss provided by the dipoles). Assume that the lattice is properly tuned for the minimum possible natural emittance.

   c. Estimate the maximum wiggler peak field allowed by the specified extracted energy spread.

   d. Assuming the wiggler peak field is the maximum allowed by the energy spread, estimate the length of damping wiggler needed to achieve the required damping times.

   e. Assuming an average horizontal beta function in the wiggler of 20 m, estimate the maximum wiggler period in order to achieve the specified extracted horizontal emittance.