## M442 Representations of Finite Groups

## Worksheet 2: Character Rules

For each of the following 5 rules find the Theorem/Proposition/Corollary where it is proven. Sometimes you must rearrange the formulas or statements in the lecture a little bit!

**Rule 1**. The sum of the squares of the degrees  $n_i$  of all irreducible representations of a finite group G is equal to the order of G:

$$\sum_{i} n_i^2 = |G|.$$

**Rule 2**. The sum of the squares of the absolute values of  $\chi(g)$  of a character  $\chi$  of an irreducible representation of a finite group G is equal to the order of the group:

$$\sum_{g} |\chi(g)|^2 = \sum_{g} \overline{\chi(g)} \cdot \chi(g) = |G|.$$

**Rule 3**. The vectors whose components are the  $\chi_i(g), \chi_j(g)$  of two characters  $\chi_i, \chi_j$  of two different irreducible representations are orthogonal:

$$\sum_{g} \overline{\chi_i(g)} \cdot \chi_j(g) = 0.$$

- **Rule 4.** The characters  $\chi(g), \chi(h)$  of two conjugate group elements are equal, where  $\chi$  is the character of a representation, irreducible or not.
- **Rule 5**. The number of irreducible representations of a group is equal to the number of conjugacy classes in the group.