

Example 1

$$H_0: \mu_1 = \mu_2 \quad \text{vs} \quad H_1: \mu_1 \neq \mu_2$$

μ_1 ... mean height of boys

μ_2 ... mean height of girls

$$\begin{aligned} S_p^2 &= \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} \\ &= \frac{9 \times 7.158^2 + 9 \times 6.859^2}{18} = 49.141 \end{aligned}$$

$$S_p = 7.0105$$

Test statistic:

$$\begin{aligned} T &= \frac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{152.82 - 159.15}{7.0105 \sqrt{\frac{1}{10} + \frac{1}{10}}} \\ &= \underline{\underline{-2.02}} \end{aligned}$$

$$t_{18}(0.05) = 1.734 < |T| \quad (\alpha = 10\%)$$

$$t_{18}(0.025) = 2.101 > |T| \quad (\alpha = 5\%)$$

We can reject H_0 at 10% level,

but not at 5% level.

$$(0.05 < p < 0.1)$$

There is weak evidence that population means of heights for boys & girls differ.

Example 2:

μ_1 mean scores before taking module
 μ_2 mean scores after taking module

$$H_0: \mu_1 = \mu_2 \quad , \quad H_1: \mu_1 < \mu_2$$

$$T = \frac{\bar{d}}{s_d / \sqrt{n}} = \frac{-10.07}{14.89 / \sqrt{15}} = \underline{\underline{-2.619}}$$

$$\bar{d} = -10.07 \quad , \quad s_d = 14.89$$

$$t_{14}(0.05) = 1.761, \quad t_{14}(0.025) = 2.145,$$

$$t_{14}(0.01) = 2.624$$

$$\text{As } -2.619 < -2.145, -2.619 > -2.624$$

We can reject at 2.5% but

We can reject at 2.5 % but
not at 1% level.

$$(0.01 < p < 0.025)$$

There is evidence that students
improved their scores after
taking the module.