

Assignment 7

Deadline = Tuesday of Week 8

Please submit written solutions in lectures, and email me your Maple file, ‘assignment7<name>.mw’, by Tuesday of Week 8.

Q1. Consider the matrix

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & -2 & 8 \end{bmatrix}$$

Use the shifted inverse power method to estimate the eigenvalue near $\gamma = -2$ and its eigenvector. Start from $z = (1, 0, 0)^T$ and write out 2 steps of the method, then use Maple to iterate the solution further and write down the solution correct to 3 decimal places. You may use the result that

$$(A + 2I)^{-1} = \begin{bmatrix} 12 & -10 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

Q2. Consider the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 10 & 0 \\ 1 & 1 & 7 \end{bmatrix}$$

Use the shifted inverse power method to estimate the eigenvalue near $\gamma = 7$ and its eigenvector. Start from $z = (0, 0, 1)^T$ and write out 1 step of the method making use of the LU factorization for $(A - 7I)$, then use Maple to iterate the solution further and write down the solution correct to 3 decimal places.

$$(A - 7I) = \begin{bmatrix} -6 & 0 & 2 \\ 0 & 3 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/6 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} -6 & 0 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

Q3. If you took maths 201 - here’s a connecting question about boundary values from the Jan 2008 exam. If you didn’t take the course, feel free to attempt the question but you won’t lose any marks if you can’t answer it.

Solve the boundary-value problem:

$$\frac{d^2y}{dx^2} + 9y = 15 \cos(2x), \quad y(0) = 3, y(\pi/2) = -1.$$

Q4. Show that the forward one-sided difference formula:

$$\frac{dy}{dx}(x_i) = y'(x_i) = \frac{y_{i+1} - y_i}{h}$$

is a first order approximation, i.e. has an error proportional to h :