Assignment 3

Deadline = Tuesday of Week 4

As previously, download the Maple file 'assignment3.mw' from VITAL in the Homework folder under sessions/resources, and save it as 'assignment3<name>.mw', where you should write your name instead of <name>. Please submit written solutions in lectures, and email me your Maple file by Tuesday of Week 4.

Q1.

Consider the linear system $Ax = b$ where

$$A = \begin{pmatrix} 1.2 & 2 & 3.4 \\ 0.8 & 1.2 & 0 \\ 0.75 & 2.3 & 1.8 \end{pmatrix}, b = \begin{pmatrix} 10 \\ 2 \\ 6 \end{pmatrix}.$$

a) Keeping at least 4 decimal digits in calculations, solve it by Gaussian elimination, and record how many algebraic operations (add, subtract, multipy, divide) you need to do. Check your answer using the Maple file.

b) Hence find the remaining entries L_{21}, D_{33} and M_{23} in the LDM decomposition of A:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ 0.625 & -7.875 & 1 \end{pmatrix} \begin{pmatrix} 1.2 & 0 & 0 \\ 0 & -0.1333 & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix} 1 & M_{12} & 2.833 \\ 0 & 1 & 17.00 \\ 0 & 0 & 1 \end{pmatrix}.$$

c) Use the LDM decomposition to solve $Ax = (0, 1, 2)^T$. Check your answer using Maple (you'll need to change the b vector to this new vector). How many algebraic operations (add, subtract, multipy, divide) did you need to do to solve this system?

Q2. Compute the Crout $A = LU$ factorization for the following tridiagonal matrix

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 3 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 5 & 2 & 4 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}.$$

Q3. Solve the linear system $Ax = b$, where

$$A = \begin{pmatrix} 0.0004 & 1.312 \\ 0.4561 & -1.894 \end{pmatrix}, \quad b = \begin{pmatrix} 1.972 \\ 1.72 \end{pmatrix}$$

a) Using Gaussian elimination without pivoting
(i) using *strictly* 4 digit arithmetic.
(ii) as accurately as your calculator will permit.
Check your answer using the Maple file.

b) Using partial pivoting strategy using *strictly* 4 digit arithmetic.

Explain why partial pivoting gives more accurate results.

Q4.

Consider the matrix A:

$$A = \begin{pmatrix} 2 & -3 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & -2 & 3 \\ 3 & 6 & 1 & 0 \end{pmatrix}.$$

Using exact arithmetic (i.e. fractions), compute the $PA = LU$ decomposition of matrix A with partial pivoting. Use the decomposition to solve the system $Ax = (4, 3, -7, 5)^T$. Check your answer using Maple.