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Feedback on Class Test

The following were the most common mistakes:

Conceptual Problems:

1) Mistakes in formulae, for example $\tau = \frac{[\gamma',\gamma'',\gamma''']}{|\gamma'\times\gamma''|^2}$, not $\frac{[\gamma',\gamma'',\gamma''']}{|\gamma'\times\gamma''|}$ or $\frac{[\gamma',\gamma'',\gamma''']}{|\gamma'|^2}$ or $\frac{[\gamma',\gamma'',\gamma''']}{|\gamma'|^3}$.

2) Confusing vectors and numbers, for example for space curves $\kappa = \frac{|\gamma'\times\gamma''|}{|\gamma'|^3}$ and not $\frac{\gamma'\times\gamma''}{|\gamma'|^3}$.

3) Confusing the formulae for the curvature of plane curves and space curves.

4) Lack of rigorous argument when showing that a curve or a surface is regular. To proof that a curve $\gamma : I \to \mathbb{R}^n$ is regular you need to show that $\gamma'(t) = 0$ for all $t \in I$. It is not enough

   - to just claim that the vector is not equal to zero everywhere,
   - to show that $\gamma'(t_0) \neq 0$ for one value $t_0 \in I$,
   - to show that $\gamma'(t_0) \neq 0$ for one value $t_0 \in I$ and claim without proof that $t = t_0$ is 'the worst possible case',
   - to say that the formulae for $\gamma'(t)$ 'involves constants' or 'involves squares'.

   See solutions to the class test for rigorous arguments in Problems 1(iii), 2(iii) and 3(ii).

Computational Problems:

1) Mistakes in $(a+b)^2 = a^2 + b^2 + 2ab$, used when computing length of vectors, for example in Problem 1(ii) the length of the velocity vector $\gamma'(t) = (1+3t^2, 2t)$ is

$$\begin{aligned}|\gamma'(t)| &= \sqrt{(1+3t^2)^2 + (2t)^2} = \sqrt{1 + (3t^2)^2 + 2\cdot 1\cdot 3t^2 + 4t^2} \\ &= \sqrt{1 + 9t^4 + 6t^2 + 4t^2} = \sqrt{1 + 10t^2 + 9t^4}\end{aligned}$$

   and not $\sqrt{1 + (3t^2)^2 + 4t^2} = \sqrt{1 + 4t^2 + 9t^4}$. Similarly in Problem 2(ii) the length of the velocity vector $\gamma'(t) = (2 + \cos t, -\sin t, 1)$ is

$$\begin{aligned}|\gamma'(t)| &= \sqrt{(2+\cos t)^2 + (-\sin t)^2 + 1^2} = \sqrt{2^2 + (\cos t)^2 + 2\cdot 2\cdot\cos t + \sin^2 t + 1} \\ &= \sqrt{4 + \cos^2 t + 4\cos t + \sin^2 t + 1} = \sqrt{6 + 4\cos t}\end{aligned}$$

   and not $\sqrt{2^2 + (\cos t)^2 + (-\sin t)^2 + 1} = \sqrt{4 + \cos^2 t + \sin^2 t + 1} = \sqrt{6}$.

2) Mistake when computing $|(X_u \times X_v)(u,v)|$ in Problem 4(i): Since $(X_u \times X_v)(u,v) = (v\cos u, v\sin u, -v)$, we have

$$\begin{aligned}|(X_u \times X_v)(u,v)| &= \sqrt{v^2\cos^2 u + v^2\sin^2 u + v^2} = \sqrt{v^2(\cos^2 u + \sin^2 u) + v^2} \\ &= \sqrt{v^2 + v^2} = \sqrt{2v^2} = |v|\cdot\sqrt{2} = v\cdot\sqrt{2}\end{aligned}$$

   and not $\sqrt{v^2 + 1}$.

3) Various sign and other mistakes when computing vector products in Problems 2(v), 2(vii), 3(ii), 4(i) and 4(iv). When working out the vector product of two vectors, it is always a good idea to check (using the dot-product) that your answer is perpendicular to each of the vectors you started with. That is, having worked out $a \times b$, check that $(a \times b) \bullet a = 0$ and $(a \times b) \bullet b = 0$.

Consult your class test to see which of these mistakes are relevant to you and which less common mistakes you have made. I hope this feedback will be helpful for your revision and exam.